JOURNAL OF CURRENT RESEARCH IN SCIENCE (ISSN 2322-5009)
CODEN (USA): JCRSDJ
2013, Vol. 1, No. 5, pp:351-355
Available at www.jcrs010.com

ORIGINAL ARTICLE

ACOUSTIC BEHAVIOR OF SINGLE UNIT IMPACT DAMPERS

Jafar Eskandari Jam[1], Aref A. Fard[1], Hossein Taghavian[2], Meghdad Khazaee[2]
1- Composite Materials and Technology Center, MUT, Tehran, Iran
2- Department of Mechanical Engineering, Buinzahra Branch, Islamic Azad University, Buinzahra, Iran

ABSTRACT: In this paper, ongoing studies to investigate the vibratory behavior of nonlinear impact dampers will be extended by considering the acoustic properties of these systems in initial design of them. For this reason, mathematical model of the sound radiated from an impact oscillator that incorporates the Hertzian contact will be analyzed. Transient sound generated from the impact of a sphere with a reflecting plane will be obtained theoretically. The calculated results can be verified with previous experimental results. By using the method of images, the acoustical noise radiated from a spherical mass colliding with a wall-floor intersection can be simulated. The theoretical results will be used to predict the sound generated by a single unit impact damper. Furthermore, a nonlinear model will be developed to investigate the effect of mass ratio, coefficient of restitution and gap size on behavior of the impact damper. Finally, damping and acoustic behavior of an impact damper will be shown in a table to find a low noise system, which can strongly suppress the undesired vibrations.
KEYWORDS: Single Unit Impact Damper, Acoustic Theory, Damping Inclination.

INTRODUCTION

One of the relatively new sources of the impact noise, as found in industry, is those caused by impact dampers. It is shown that the impact dampers can operate more efficiently than classical dynamic vibration dampers (Blazejczyk-Okolewska, 2001). Impact dampers can be extensively applied to attenuate undesired vibration of robot arms, turbine blades and so on (Zhang and Angeles, 2005). Behavior of impact dampers have been investigated experimentally, analytically and numerically for many years (Cheng and Xu, 2006; Afsharfard and Farshidianfar, 2012; Afsharfard and Farshidianfar, 2013; Bapat and Sankar, 1985). It is shown that performance of an impact damper in free damped vibration is strongly depends on the clearance between the impact mass and main mass (gap size). Cheng and Wang, (2003) indicated that the gap should be smaller than twice of the initial displacement of the main mass in free damped vibrations. Coefficient of restitution and the mass ratio have great effects on the application of impact dampers (Bapat and Sankar, 1985). Cheng and Xu, (2006) obtained a relation between coefficient of restitution and impact damping ratio. The main target of the present work is to analyze the nonlinear behavior of a vibratory system equipped with an impact damper and optimize the results with the acoustical considerations.

The classical theory of the acoustical noise radiated from vibratory bodies has been developed in detail for the harmonic steady-state case (Fahy and Gardonio, 2007). Kirchhoff derived the radiation field generated by an impulsive translational acceleration of a sphere. Koss and Alfredson, (1973) inferred the functional relation between the sound pressure of the impact of balls and time. Akay and Hodgson, (1978) presented a theoretical derivation of the sound pressure waveform for an elastic impact of a sphere with a massive slab. Yufang and Zhongfang, (1992) investigated the radiation of acoustical noise from impact of two cylinders. In the presented work, for defining the force acting between the mass colliding with barrier, the classical Hertzian impact theory is used. The variable coefficient of restitution, which varies with the mass ratio, is achieved analytically using the Newmark-beta integration method. The analytically calculated answers are compared and verified using the previous exact results. Furthermore, to improve the initial design of the impact dampers, the acoustical behavior of these systems is accounted.

MECHANICS OF IMPACT

In the presented work, the Hertz theory is applied to describe the contact of a sphere with a reflecting plane. The Hertzian contact is based on the assumption that the elastic energy acquired by the colliding bodies during impact is

---

Corresponding Author: Meghdad Khazaee, Department of Mechanical Engineering, Buinzahra Branch, Islamic Azad University, Buinzahra, Iran. **E-mail:** MeghdadKhazaee@gmail.com

entirely reversible. For an elastic impact, the body deformation is localized and the corresponding energy losses can be assumed negligible. The expression for the contact force according to the Hertzian model can be written as follows (Goldsmith, 2002; Ibrahim, 2009):

$$F_{HZ} = K_{HZ}\alpha^{3/2} \qquad (1)$$

Where; α is the relative displacement of colliding bodies (value of the approach) and K_{HZ} is the Hertzian contact stiffness which can be obtained using the following relation:

$$K_{HZ} = \frac{4}{3\pi}\frac{\sqrt{a}}{(1-\nu_1^{\,2})/\pi E_1 + (1-\nu_2^{\,2})/\pi E_2} \qquad (2)$$

Where; ν is the Poisson's ratio, E is the Young's modulus, a is the radius of the sphere and subscripts 1 and 2 refer to the sphere and the slab, respectively (Akay and Hodgson, 1978). According to the Hertzian contact and the Newton's second law of motion, the maximum acceleration of the impacting mass is equal to:

$$A_{\max} = \frac{K_{HZ}}{m}\left(\frac{5U_0^{\,2}}{4K^{*}K_{HZ}}\right)^{3/5} \qquad (3)$$

Where; U_0 is the contact velocity amplitude, m is the mass of the colliding sphere and $K^{*} = 1/m$. Hence, the impact acceleration of the mass is calculated as:

$$A(t) = A_{\max}\sin\left(\frac{\pi t}{\tau}\right) \qquad (4)$$

Where; τ is the contact duration and it is equal to $\tau = 2.9432\alpha_{\max}/U_0$ (Yufang and Zhongfang, 1992).

ACOUSTIC THEORY

Consider, a sphere travels with an initial velocity collides with a reflecting wall. For the impact situation, the centre of the sphere was chosen as the centre of a spherical coordinate system. The sound field can be described by (Wilson, 2005):

$$u = \frac{\partial\phi}{\partial r} \qquad (5)$$

Where; u is particle velocity and ϕ is a potential velocity. The wave equation is as follows:

$$\frac{\partial^2(r\phi)}{\partial r^2} = \frac{1}{c^2}\frac{\partial^2(r\phi)}{\partial t^2} \qquad (6)$$

Where; c is the wave propagation velocity. The velocity potential generated by an oscillating sphere of radius a is shown in following relation (Koss and Alfredson, 1973):

$$\phi(r,t,\theta) = \frac{a^3 v}{r^2}\frac{(1+jkr)\cos\theta}{2(1+jka)-(ka)^2}\exp\left(j\omega t - jk(r-a)\right) \qquad (7)$$

Where; k is the wave number and it is equal to ω/c, ω is the angular frequency of oscillation, θ is the polar angle, j is the positive root of minus one and v is the velocity amplitude . The velocity of the sphere due to a unit impulse of acceleration can be written as follows:

$$v(\omega) = \pi\delta(\omega) + \frac{j}{\omega} \qquad (8)$$

Where; δ is Dirac delta function. The velocity potential for an arbitrary velocity $v(\omega)$ can be achieved, using Fourier synthesis (Morse, 1981), as follows:

$$\phi(r,t,\theta) = \frac{a^3\cos\theta}{r^2}\int_{-\infty}^{+\infty}\frac{v(\omega)(1+jkr)\exp\left(j\omega t - jk(r-a)\right)}{2(1+jka)-(ka)^2}d\omega \qquad (9)$$

The solution of the above relation is evaluated using contour integration in the complex plane. It is shown as follows:

$$\phi(r,t,\theta) = \frac{a^3\cos\theta}{2r^2}\left\{\left[\left(\frac{2r}{a}-1\right)\sin(lt') - \cos(lt')\right]\exp(-lt') + 1\right\} \qquad (10)$$

Where; $t' = t - (r-a)/c$ and $l = c/a$.

Therefore, the acoustic pressure, which is equal to $p = \rho_0\left(\partial\phi/\partial t'\right)$, can be written as follows:

$$p = \rho_0 c\frac{a^2\cos\theta}{r^2}\left\{\left(1-\frac{r}{a}\right)\sin(lt') + \left(\frac{r}{a}\right)\cos(lt')\right\}\exp(-lt') \qquad (11)$$

The acoustic pressure radiated from a sphere subjected to an arbitrary acceleration $A(t)$ can be calculated by the convolution technique (Wilson, 2005) where:

$$p = \int_0^B p(t'-\xi)A(\xi)d\xi \qquad (12)$$

The ξ in the above relation is the integration variable.

$$p = \left(\rho_0 c\frac{a^2}{r^2}\cos\theta\right)\left(\frac{125}{64}\frac{K_{HZ}^2 U_0^6}{m^2}\right)^{1/5}\int_0^B\{(1-r/a)\sin\left(l(t'-\xi)\right) + (r/a)\cos\left(l(t'-\xi)\right)\}\exp\left(-l(t'-\xi)\right)\sin\left(\frac{\pi\xi}{d}\right)d\xi \qquad (13)$$

The above integral should be evaluated for two different time intervals because the Hertzian acceleration exists for only a finite period. Presence of the reflecting plane can be accounted by employing the method of images

(Crocker, 2007). The resulting pressure at the field point P is then:

$$p(r,\theta,t) = p(r_1,\theta_1,t_1) - p(r_2,\theta_2,t_2) \quad (14)$$

Parameters r_1, r_2, θ_1 and θ_2 are shown in Figure 1.

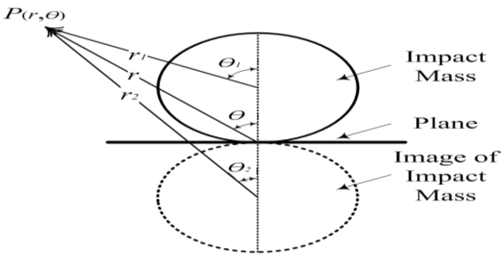

Figure 1: Dipole and its image representation of the ball reflecting plane impact problem.

A calculation of the pressure waveform for the impact of spherical mass with barrier is shown in Figure 2. Furthermore, for verifying, previous result (Akay and Hodgson, 1978) is illustrated in this figure. This waveform is characteristically M-shaped, with a larger negative pressure peak than the two positive values.

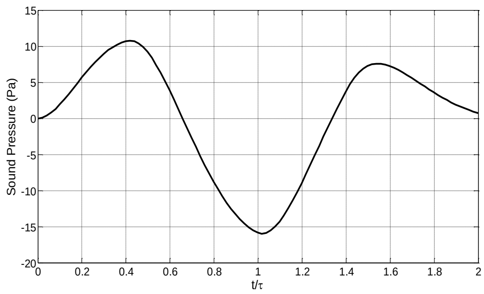

Figure 2: Sound pressure wave for the impact of a sphere with reflecting plane ($r = 0.7m$, $\theta = \pi/4$, $a = 19.05mm$, $U_0 = 1.4m/s$ and $K_{HZ} = 5.6\times10^8 N/m^{1.5}$).

The sound pressure level of the acoustic noise radiated from the dipole is calculated circumferentially around the contact point. The directivity pattern for the dipole source is shown in Fig. 3.

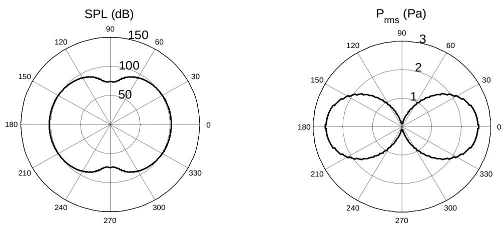

Figure 3: Theoretical directive SPL (A) and root mean square pressure (B) radiated from the dipole.

SINGLE UNIT IMPACT DAMPER

An impact damper consists of a small loose mass within a main mass that freely moves through an enclosure to suppress undesirable vibrations. The energy loss in impact dampers depends on effective impacts. The dynamic characteristics of the impact dampers can be analyzed using two-degrees-of-freedom models as shown in Figure 4. This model is consisted of an oscillator with linear stiffness K, mass M, viscous damping C and an impact damper with mass m and gap size d.

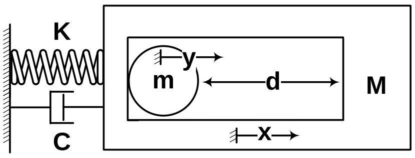

Figure 4: Schematic figure of a vibratory system equipped with an impact damper.

When the impact mass collides with the main mass, an impulsive force acts on both of them. In this work, the Hertzian contact model presents the impact between the masses. Consequently, when the impact mass collides with the main mass, the differential equations of motion can be written as follows:

$$\begin{aligned} &M\frac{d^2x}{dt^2} + C\frac{dx}{dt} + Kx = c_1\frac{dz}{dt} + K_{HZ}z^{3/2} \\ &m\frac{d^2y}{dt^2} + c_1\frac{dz}{dt} + K_{HZ}z^{3/2} = 0 \end{aligned} \quad (15)$$

Where; $z=y-x$ is relative displacement and c_1 is equivalent contact damping constant. The values of the model parameters, in this work, are listed in the Table 1.

Table 1: Parameters for the vibratory system with an impact damper

$K = 500$ N/m	$C = 0.05$ N.S/m
$K_{HZ} = 5.6\times10^8$ N/m$^{1.5}$	$c_1 = 20$ N.S/m
$m = 40.5$ gr	$M = 0.5$ Kg

The Newmark-beta integration method is used to solve Eq. (15), analytically (Clough and Penzien, 2010). The time step, in this work, is considered as $\Delta t = 10^{-7}$ sec . The waveforms of free vibrations without and with impact damper are shown in Figure 5.

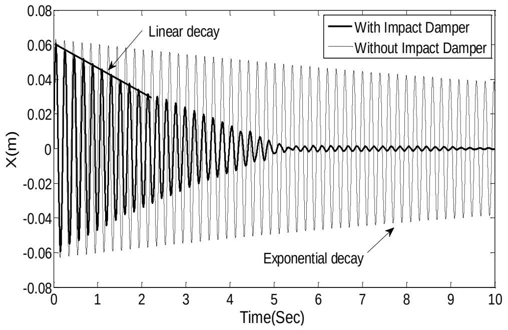

Figure 5: Waveforms of free vibrations without and with impact damper ($\dot{x} = 2$ m/s and $d = 4$ cm).

The initially linear decrease in the maximum displacement of the vibratory system, with impact dampers, is usually represented by damping inclination, which is defined as:

$$e = \frac{X_1 - X_2}{t_2 - t_1} \qquad (16)$$

Where; t_1 and t_2 are the times of occurrence of the maximum positive displacements X_1 and X_2 respectively. Variation of the coefficient of restitution with the impact mass and contact velocity, for the presented vibratory system, is shown in Fig. 6. As shown in this figure, the damping inclination does not vary uniformly by changing the impact mass and the gap size. Unlike the damping inclination, the maximum sound pressure level varies relatively uniform with changing the impact mass and the gap size. The uniform varying of the maximum sound pressure level is shown in the following figure.

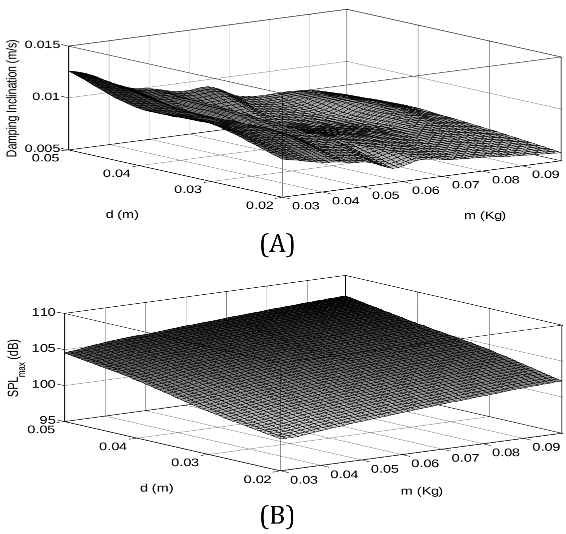

Figure 6: Variation of the damping inclination with the gap size and the impact mass (A); variation of the maximum sound pressure levels with impact mass and gap size (B).

DESIGN OF IMPACT DAMPERS REGARDING TO ACOUSTIC AND VIBRATORY BEHAVIOR

Unlike usual method to investigate impact dampers, in this work, the coefficient of restitution varies with the mass ratio. Variation of the coefficient of restitution versus the impact mass and the corresponding fitted curve are shown in Figure 7.

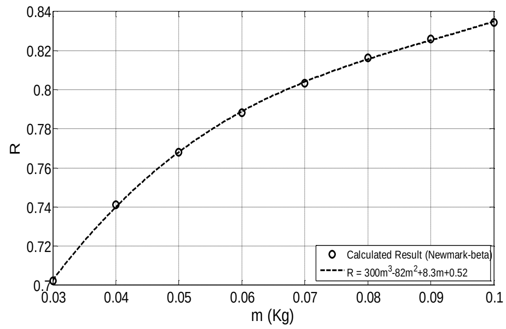

Figure 7: Variation of the coefficient of restitution versus the impact mass (M=0.5Kg).

In this work, it is claimed that the design of an impact damper should be divided to two parts. They can be conveniently called as ″vibration-based″ and ″acoustic-based″ designs. Extent of the maximum SPL in each value of the damping inclination is shown in Table 2. Regarding to the tabulated data, it can be concluded that if the damping inclination varies between 0.7cm/s and 1.2cm/s, the acoustic-based consideration can decrease the sound pressure level between 2dB and 8dB. As shown in Table 2, if the damping inclination varies between the usual range 0.8cm/s and 1.0cm/s, the acoustic-based design, works very good and decreases the sound pressure level more than 7%.

Table 2: Extent of the maximum SPL in each value of the damping inclination

e(cm/s)	Minimum of SPL_{max} (dB)	Maximum of SPL_{max} (dB)	Decrease in SPL_{max} (dB)
0.7	103	105	2
0.8	100	108	8
0.9	100	107	7
1.0	100	107	7
1.1	101	106	5
1.2	104	106	2

CONCLUSION

An analytical model has been used to predict the sound pressure waveform radiated from the impact of a spherical mass with a reflecting plane. Moreover, the dynamic behavior of the vibro-impact system is presented using a nonlinear model that incorporates the Hertzian contact theory. The nonlinear differential equations are solved by the Newmark-beta method. It is clearly shown that the analytical solutions are in close agreement with the previous exact results.

Effects of varying the mass ratio and the gap size have been studied on the dynamic and acoustic behavior of a vibratory system equipped with an impact damper. It is shown that, unlike the damping inclination, the maximum sound pressure level that is generated by the impact damper, varies uniformly with changing the gap size and the mass ratio.

Finally, it is shown that for a relatively extensive range of the damping inclination, the acoustic-based consideration decreases the sound pressure level up to 8dB.

REFERENCES

Afsharfard A, Farshidianfar A. Design of nonlinear impact dampers based on acoustic and damping behavior. International Journal of Mechanical Sciences 2012;65(1):125-133.

Afsharfard A, Farshidianfar A. Free vibration analysis of nonlinear resilient impact dampers. Nonlinear Dynamics 2013;pp:1-12.

Akay A, Hodgson TH. Acoustic radiation from the elastic impact of a sphere with a slab. Applied Acoustics 1978;11(4):285-304.

Bapat CN, Sankar S. Single unit impact damper in free and forced vibration. Journal of Sound and Vibration 1985;99(1):85-94.

Blazejczyk-Okolewska B. Analysis of an impact damper of vibrations. Chaos, Solitons and Fractals 2001;12(11):1983-1988.

Cheng CC, Wang JY. Free vibration analysis of a resilient impact damper. International Journal of Mechanical Sciences 2003;45(4):589-604.

Cheng J, Xu H. Inner mass impact damper for attenuating structure vibration. International Journal of Solids and Structures 2006;43(17):5355-5369.

Clough RW, Penzien J. Dynamics of structures. 2nd Ed., Berkeley, Calif., Computers and Structures 2010;pp:738.

Crocker MJ. Handbook of noise and vibration control. In: Hoboken NJ. John Wiley 2007;pp:1569.

Fahy F, Gardonio P. Sound and structural vibration : radiation, transmission and response. 2nd Ed. Amsterdam, Boston: Elsevier 2007;pp:633.

Goldsmith W. Impact: the theory and physical behaviour of colliding solids. In: Mineola NY. Dover Publications 2002;pp:379.

Ibrahim RA. Vibro-impact dynamics: modeling, mapping and applications. Lecture notes in applied and computational mechanics. Berlin: Springer Verlag 2009;pp:297.

Koss LL, Alfredson RJ. Transient sound radiated by spheres undergoing an elastic collision. Journal of Sound and Vibration 1973;27(1):59-68.

Morse PM. Vibration and sound. New York: American Institute of Physics for the Acoustical Society of America 1981;pp:468.

Wilson CE. Noise control. Rev Ed., Malabar, FL: Krieger 2005.

Yufang W, Zhongfang T. Sound radiated from the impact of two cylinders. Journal of Sound and Vibration 1992;159(2):295-303.

Zhang D-G, Angeles J. Impact dynamics of flexible-joint robots. Computers and Structures 2005;83(1):25-33.