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ORIGINAL ARTICLE

OPTIMUM STACKING SEQUENCE OF SYMMETRIC COMPOSITE LAMINATES FOR MINIMUM RESIDUAL STRESSES

Mohammad Hadi Hajmohammad[1], Taghi Sharafi[2], Seyed Sadegh Mirtalebi Esforjani[2]
1- Department of Mechanical Engineering, Imam Hossein University, Tehran, Iran
2- Department of Mechanical Engineering, Najafabad Branch, Islamic Azad University, Isfahan, Iran

ABSTRACT: Residual stresses are self equilibrating stresses that a portion of the strength of the specimen is used to oppose these stresses. Classical Laminate Plate Theory (CLPT) can be used for calculating of these stresses. In this paper, genetic algorithm is introduced to optimization of stacking sequence. We treat with this problem as Traveling Salesman Problem (TSP). Genetic algorithm (GA) approaches are successfully implemented to the TSP. The residual stresses of a constant thickness composite plate, which is obtained by the analytical method, was used as the fitness function in the GA to find its minimum value by arrange the ply stacking sequence. Two examples of constant thickness and symmetrical composite laminate plates were used to examine the performance of the proposed algorithms. The results of computational tests presented are very encouraging.
KEYWORDS: Stacking Sequence, Genetic Algorithms, Optimization, Composite Laminate, Residual Stress.

INTRODUCTION

Residual stresses are self-equilibrating stresses that are trapped in a specimen even if the specimen is not under external loads. Therefore, a portion of the strength of the specimen is used to oppose these stresses. In metals, residual stresses exist due to local yielding, welding, and melting (Shokrieh and Ghasemi, 2007a). In composites, due to mismatch of the coefficient of thermal expansion of fibers and resin, residual stresses are produced during the manufacturing process. Cure temperature plays a key role in determining the sizes of the residual stresses. The magnitudes of these stresses depend on the ply lay-up, material properties of the unidirectional ply, and manufacturing processes. These residual stresses reduce the efficiency of structures and can cause matrix cracking, fiber breakage, and delamination, thus reducing the ultimate strength of structures (Shokrieh and Ghasemi, 2007b).

Fiber reinforced composite laminated structures have superior characteristics in design, i.e. variable stacking sequence and ply angles, when compared to conventional materials. Therefore, optimal design can be achieved by determining the proper stacking sequences and the number of plies. However, there are some difficulties in optimal design such as discreteness of the design values and complexity of the design space. Moreover, it was inevitable to use discrete ply angles such as 0º, 90º, or ±45º for designing realistic composite structures. However, in the early 1990s, continuous ply angles were considered (Adali and Duffy, 1990). From the mid-1990s, there was more interest in the use of discrete ply angles, and some researchers used discrete optimization techniques (Adali *et al.*, 1995; Adali *et al.*, 1996). One of the discrete optimization techniques, Genetic Algorithm was proposed for the optimization of composite structures, and many researchers reported that Genetic Algorithm was a good solution for complex optimization problems such as composite structures (Riche and Haftka, 1993; Malott *et al.*, 1996; Yamazaki, 1996; Nagendra *et al.*, 1996).

Residual stress minimization is the final goal for this paper The GA is an optimization technique based on a simulation of Darwin's theory of life evolution. From a randomized population, the GA evolves using three operators based on several fit goals in order to get an acceptable population. The advantage of GA's is that they give the designer a family of near optimal designs with a small variation in their performance index instead of a single design. The design of composite laminates has often been formulated as a continuous optimization problem, with ply thickness and ply orientation angles as design variables. However, for many practical problems, ply thicknesses are fixed and ply orientation angles are limited to a small set of angles, such as 0º, 90º and ±45º. The design of the laminate then becomes a stacking sequence optimization problem, which can be formulated

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Corresponding Author: Seyed Sadegh Mirtalebi Esforjani, Department of Mechanical Engineering, Najafabad Branch, Islamic Azad University, Isfahan, Iran. **E-mail:** nanofluidmehanics@gmail.com

as an integer programming problem. Interest in the application of genetic algorithms (GA) to such integer programming problems has increased over recent years.

CALCULATION OF RESIDUAL STRESSES

In residual stress calculations using the classical lamination theory (CLT), the following Assumptions are made:

- The composite lamina is thin; therefore it is under plane stress condition.
- Material behavior is linear elastic and does not vary with time.
- This method calculates the macroscopic residual stresses.

As the shrinkage is different for different layers, in cooling from curing temperature to the room temperature, they are under tension and compression and therefore, thermal loads are induced. To calculate the residual stresses, these thermal loads must be calculated (Shokrieh and Kamali, 2005).

If the thermal forces and the moment resultants are defined as $N^{*T} = \left(N_x^T, N_y^T, N_{xy}^T\right)$, $M^{*T} = \left(M_x^T, M_y^T, M_{xy}^T\right)$ then the strain and the curvature of the mid plane are calculated as follows:

$$\varepsilon^{\circ} = \alpha . N^{*} + \beta . M^{*} \quad (1)$$

$$k^{\circ} = \beta^{T} . N^{*} + \delta . M^{*} \quad (2)$$

Where $\alpha, \beta \; and \; \delta$ matrices are calculated using the extensional, coupling, and bending stiffness matrices (A, B, and D respectively) as follows:

$$\alpha = A^{-1} + A^{-1} B D^{*-1} B A^{-1} B \quad (3)$$

$$\beta = -A^{-1} B D^{*-1} \quad (4)$$

$$\delta = D^{*-1} \quad (5)$$

$$D^{*} = D - B A^{-1} B \quad (6)$$

Studies on Residual Stresses in Polymer Composites sing the strain and the curvature of the mid plane; the residual strains and stresses of each layer in the off-axis coordinate system are calculated as follows:

$$\varepsilon_r^{(k)} = \left(\varepsilon^{0} + \bar{z}_k k^{0} - \alpha^{(k)} \Delta T\right) \quad (7)$$

$$\sigma_r^{(k)} = \bar{Q}^{(k)} \varepsilon_r^{(k)} \quad (8)$$

2.1. The Genetic Algorithms

The Genetic Algorithm (GA) is a stochastic global search method that mimics the metaphor of natural biological evolution. GA operates on a population of potential solutions applying the principle of survival of the fittest to produce (hopefully) better and better approximations to a solution. At each generation, a new set of approximations is created by the process of selecting individuals according to their level of fitness in the problem domain and breeding them together using operators borrowed from natural genetics. This process leads to the evolution of populations of individuals that are better suited to their environment than the individuals that they were created from, just as in natural adaptation. The *fitness function* establishes the basis for selection of pairs of individuals that will be mated together during reproduction.

During the reproduction phase, each individual is assigned a fitness value derived from its raw performance measure given by the objective function. This value is used in the selection to bias towards more fit individuals. Highly fit individuals, relative to the whole population, have a high probability of being selected for mating whereas less fit individuals have a correspondingly low probability of being selected.

Once the individuals have been assigned a fitness value, they can be chosen from the population, with a probability according to their relative fitness, and recombined to produce the next generation. Genetic operators manipulate the characters (genes) of the chromosomes directly, using the assumption that certain individual's gene codes, on average, produce fitter individuals. The recombination operator is used to exchange genetic information between pairs, or larger groups, of individuals. Flowchart 1 shows the steps of GA (Gen and Cheng, 1997; Potvin, 1996; Houck *et al.*, 1996).

2.2. Stacking Sequence of Composite Laminates

In this problem a sequence is desired that has minimum Residual Stresses between layers. In this paper we treat with this problem as Traveling Salesman Problem (TSP). TSP is a typical example of a very hard combinatorial optimization problem. Consider a graph shows some city and rods among them. In this graph, *G= (V, E)*, V is set of vertices that each vertex is a city. The edge set E of graph are relation among cities. The problem is to find the shortest tour that passes through each vertex in a given graph exactly once. Tour is a complete permutation of vertices such that two successive city in this permutation can be connected by an edge. If you consider a complete graph, then all permutation of nodes are possible tours.

In our problem, there is a sequence or permutation of Composite laminates with minimum Residual Stresses in X-, Y- and XY-direction. In both of problems, search algorithm has to find a permutation that has minimum value of a fitness function. Consider a permutation P that shows the sequence of laminate. Fitness Function in GA has to calculate the residual stresses according to P and return a single value. More recently, GA approaches are successfully implemented to the TSP (Gen and Cheng, 1997). Potvin presents survey of GA approaches for the general TSP (Potvin, 1996).

2.3. Application of GA for minimization of Residual Stresses

We have to identify some aspects of GA for applying on our problem: chromosome representation, crossover operator, mutation operator, selection method. Before implementation some parameters must be configured: population size, crossover rate, mutation probability, etc. The integer array is used to encoding a chromosome in GA. This array has to be a permutation of 1 to n. n is the number of laminate. For making cross between two chromosomes, cyclic crossover is used (Houck *et al.,* 1996). In this crossover a child inherent data on a cycle from first parent and complete by second parent. Swap mutation is used as mutation operator (Houck *et al.,* 1996). This operator swaps two elements of a chromosome and produces mutated chromosome.

NUMERICAL RESULTS

Numerical results are given for glass/epoxy laminate to show minimization of residual stress with GA. This minimization is applied for nine ply layers and thirty ply layers. The ply configurations of specimens are as follows:

(1) [10/20/30/40/50/60/70/80/90]

(2) [3/6/9/12/15/18/21/24/27/30/33/36/39/42/45/48/51/54/57/60/63/66/69/72/75/78/81/84/87/90]

The mechanical properties of glass/epoxy are shown in Tables 1.

Table 1: elastic constants of glass/epoxy composites

E_x (GPa)	E_y (GPa)	G_{xy} (GPa)	υ_{xy}	α_x (μ/°C)	α_y (μ/°C)
34.8	10.0	3.55	0.237	5.4	26

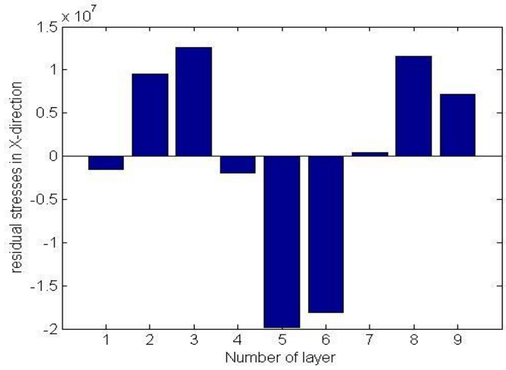

FIG.A.1: σ_{R_x} for nine layers

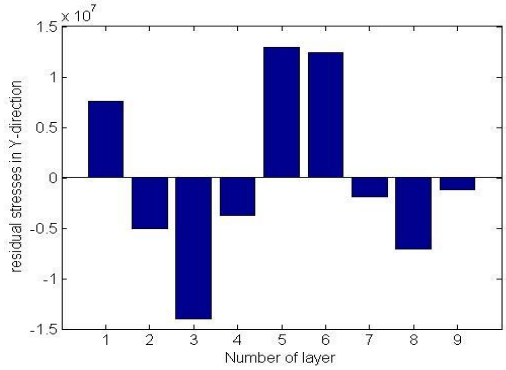

FIG.A.2: σ_{R_y} for nine layers

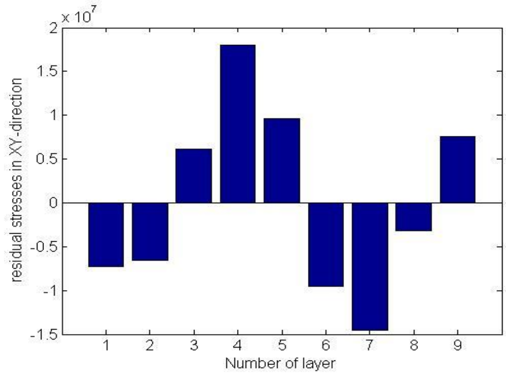

FIG.A.3: $\sigma_{R_{xy}}$ for nine layers

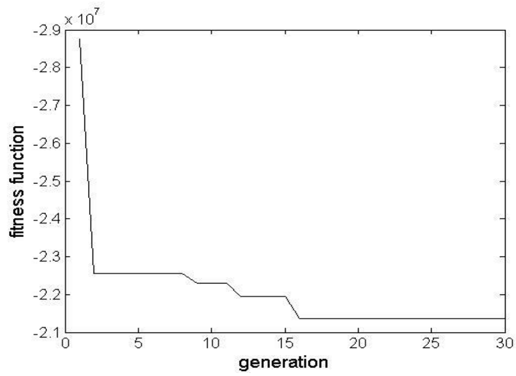

FIG.B: value of fitness function in per generation

The optimization result showed that the optimal stacking sequence for first configuration is [50/70/60/10/80/30/90/40/20] and for a second configuration is [30/36/81/39/75/84/33/12/6/69/60/72/57/24/42/66/45/78/54/9/51/15/18/87/63/21/3/48/90/27].

The result of residual stress for configuration 1 has shown in figure (A). In accordance with this figure the maximum deference between maximum and minimum residual stress has given in shear stress by figure A.3. By GA optimization the fitness function has optimized and the result of fitness function in per generation has shown in figure (B). The residual stress for nine layers after minimization has shown in figure (C). In accordance with this figure the maximum deference between maximum and minimum residual stress is in x-direction and y-direction have represented by figure C.1 and C.2.

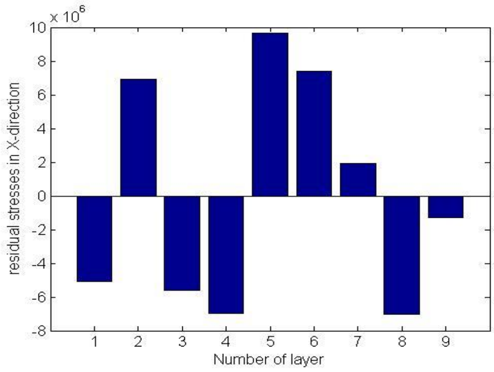

FIG.C.1: σ_{R_x} for nine layers

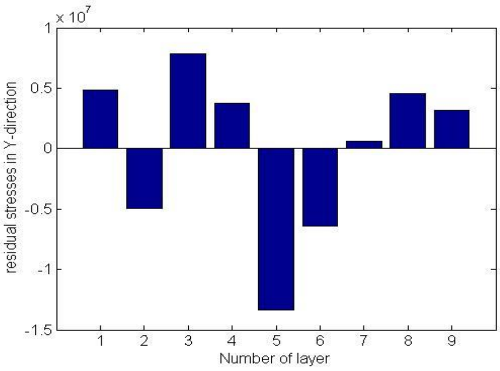

FIG.C.2: σ_{R_y} for nine layers

The result of the residual stress for configuration 2 with thirty layers has shown in figure (D). In accordance with this figure the maximum deference between maximum and minimum residual stress is in x-direction and has represented by figure D.1.

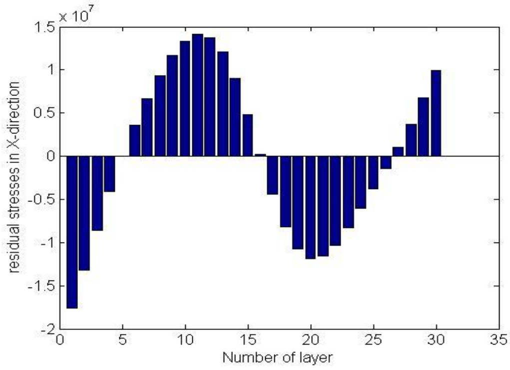

FIG.D.1: σ_{R_x} for thirty layers

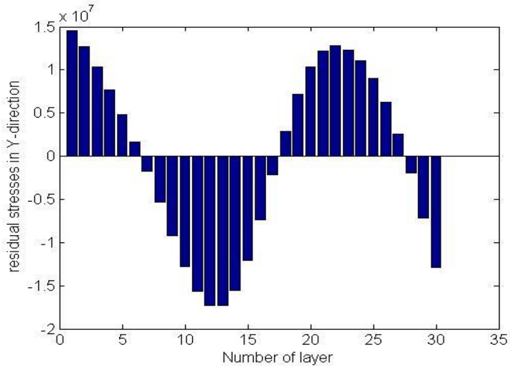

FIG.D.2: σ_{R_y} for thirty layers

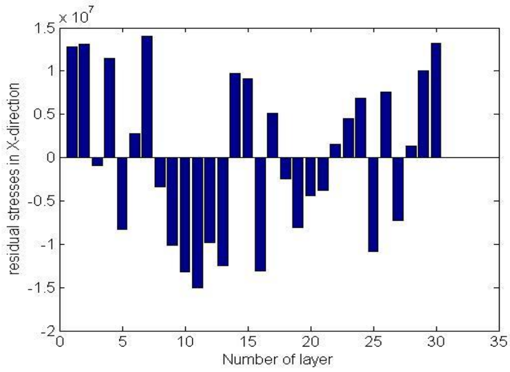

FIG.E.1: σ_{R_x} for thirty layers

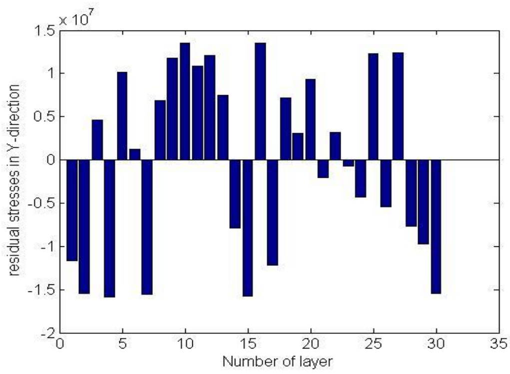

FIG.E.2: σ_{R_y} for thirty layers

The residual stress for thirty layers after minimization has shown in figure (E). In accordance with this figure the maximum deference between maximum and minimum residual stress is in x-direction and has represented by figure E.1.

CONCLUSION

In this research, genetic algorithm is introduced to optimization of stacking sequence for reduction of residual stresses in composite laminates. Numerical results show that genetic algorithm can be used for these goals. For a laminate with thirty plies residual stresses reduced up to 20%. For a laminate with eight plies, residual stresses reduced in X direction about 60%, in Y direction about 40% and in XY direction about 30%. These results are important and valuable.

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