

**NUMERICAL SIMULATION OF MIXED CONVECTION IN TWO-SIDED LID DRIVEN SHALLOW CAVITY
SUBJECTED TO NANOFLUID; IMPACT OF VELOCITY RATIO AT SPECIFIC RICHARDSON NUMBER**

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ABSTRACT: In this study, mixed convection flow and heat transfer in a two-sided lid-driven shallow cavity with different velocity ratio subjected to Al_2O_3 -water nanofluid with mean particle diameter of 80 nanometers is studied numerically using the finite volume method. Thermal conductivity and effective viscosity of nanofluid were determined using Hamilton-Crosser and Jang models, respectively. The influence of different velocity ratios at specific Richardson number of the nanofluid, on hydrodynamic and thermal characteristics is discussed. The results indicated that increase in velocity ratio enhances heat transfer.

KEYWORDS: Nanofluid, Heat Transfer, Mixed Convection, Shallow Cavity, Richardson Number.

INTRODUCTION

Low conduction heat transfer which leads to reduced heat transfer rate is among one of the greatest challenges in the field of cooling fluid such as water and ethylene glycol and cte. Therefore, in order to enhance conduction heat transfer for these cooling fluids techniques such as changing flow geometry, boundary conditions, and enhanced fluid thermophysical properties can be used. Among these techniques is the process of adding particles in the scale of nano to the fluid which causes the heat transfer to increase. Many researchers studied natural convection of nanofluids in simple rectangular enclosures by considering various boundary conditions: [Muthamilselvan et al., \(2010\)](#) studied the mixed convection heat transfer in a lid-driven rectangular enclosure filled with the Copper-water nanofluid. In their study, the side walls of the enclosure were assumed to be insulated while its horizontal walls were kept at constant temperatures, and the top wall was also considered to be moving at a constant velocity. They observed that both the cavity aspect ratio and the volume fraction of nanoparticles affected the fluid flow and heat transfer inside the enclosure. In another work, [Khanafer et al., \(2003\)](#) presented a two dimensional numerical simulation of natural convection of nanofluids inside a vertical rectangular enclosure. [Sheikhzadeh et al., \(2011\)](#) have carried out a numerical simulation to investigate free convection of Cu-water nanofluid inside a square cavity with partially thermally active side walls. The active parts of the left and the right

side walls of the cavity were maintained at hot and cold state, respectively, while the cavity's top and bottom walls, as well as the inactive parts of the side walls were assumed to be insulated. They found that maximum value of average Nusselt number for high Rayleigh numbers occurred when the hot and cold parts were positioned in the bottom and middle regions of the vertical walls, respectively. Also there are other studies on natural convection of nanofluids in non-square cavities.

Also, there are various numerical studies investigating the effects of nanofluid on combined natural and forced convection heat transfer (mixed convection heat transfer) in the cavities. [Tiwari and Kumar, \(2007\)](#) were the first authors to carry out a numerical study on mixed convection flow and heat transfer of Cu-water nanofluid inside a square cavity with insulated top and bottom walls and differentially-heated moving sidewalls. They concluded that when Richardson number is equal to unity, the average Nusselt number had a substantial increase with increase in the volume fraction of the nanoparticles. [Ghasemi and Aminossadati, \(2010\)](#) also investigated mixed convection fluid flow and heat transfer of Al_2O_3 -water nanofluid inside a right triangular cavity with insulated horizontal wall, hot inclined wall and moving cold vertical wall. They concluded that adding nanoparticles increases the rate of heat transfer for all values of Richardson number and for each direction of the sliding wall movement.

Recently, a few investigation of mix convection fluid flow and heat transfer of nanofluid in a lid

driven cavity have been done using variable properties formulation ([Sadodin et al., 2011](#); [Fereidoon et al., 2013](#); [Zarei et al., 2013](#); [Abbasian Arani et al., 2012](#); [Sadodin et al., 2013](#); [Hemat Esfe et al., 2012](#); [Ghadi et al., 2013](#); [Hemmat Esfe and Sadodin, 2012](#); [Sadodin and Hemmat Esfe, 2012](#)).

Current study is focused on the mixed convection in a square lid-driven inclined cavity with a hot rectangular obstacle and subjected to Al₂O₃-water nanofluid based on recent variable properties model.

MATHEMATICAL MODELING AND FORMULATION

Fig. 1 shows a two-dimensional square cavity considered for the present study with physical dimensions. The cavity is subjected to Al₂O₃-water nanofluid and having adiabatic vertical moving lids. Top wall is kept at low temperature (T_c) while the right wall is maintained in hot temperature (T_h). The height and the width of this obstacle are denoted by *H* and *L*. velocity of left moving lid denoted by U₀ while right lid move with U₀*V.R constant speed.

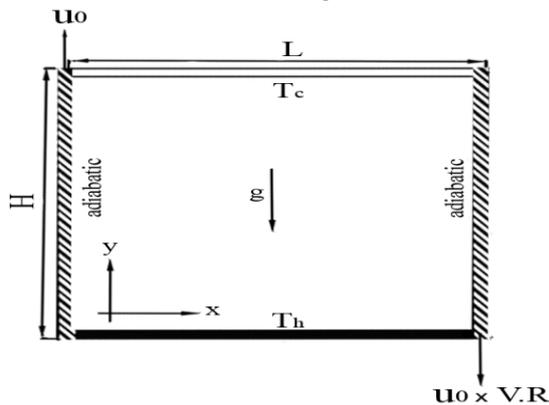


Figure 1: Schematic diagram of current problem

The nanofluid in the enclosure is Newtonian, incompressible and laminar. In addition, it is assumed that both the fluid phase and nanoparticles are in the thermal equilibrium state and they flow with the same velocity. The density variation in the body force term of the momentum equation is satisfied by Boussinesq's approximation. Also, in this study variable properties formulation is used for nanoparticle with 80 nanometers and R=0.007.

The thermal conductivity and the viscosity of the nanofluid are taken into consideration as variable properties; both of them change with volume fraction and temperature of nanoparticles. Under the above assumptions, the system of governing equations is:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{1}$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = -\frac{1}{\rho_{nf}} \frac{\partial p}{\partial x} + \nu_{nf} \nabla^2 u + \frac{(\rho\beta)_{nf}}{\rho_{nf}} g \Delta T \sin(\gamma) \tag{2}$$

$$u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} = -\frac{1}{\rho_{nf}} \frac{\partial p}{\partial y} + \nu_{nf} \nabla^2 v + \frac{(\rho\beta)_{nf}}{\rho_{nf}} g \Delta T \cos(\gamma) \tag{3}$$

And

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha_{nf} \nabla^2 T. \tag{4}$$

The dimensionless parameters may be presented as

$$X = \frac{x}{L}, Y = \frac{y}{L}, V = \frac{v}{u_0}, U = \frac{u}{u_0} \tag{5}$$

$$\Delta T = T_h - T_c, \theta = \frac{T - T_c}{\Delta T}, P = \frac{p}{\rho_{nf} u_0^2}.$$

Hence,

$$Re = \frac{\rho_f u_0 L}{\mu_f}, Ri = \frac{Ra}{Pr \cdot Re^2}, Ra = \frac{g \beta_f \Delta T L^3}{\nu_f \alpha_f}, Pr = \frac{\nu_f}{\alpha_f} \tag{6}$$

The dimensionless form of the above governing equations (1) to (4) become

$$\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0 \tag{7}$$

$$U \frac{\partial U}{\partial X} + V \frac{\partial U}{\partial Y} = -\frac{\partial P}{\partial X} + \frac{\nu_f}{\nu_{nf}} \frac{1}{Re} \nabla^2 U + \frac{Ri}{Pr} \frac{\beta_f}{\beta_{nf}} \Delta \theta \sin(\gamma) \tag{8}$$

$$U \frac{\partial V}{\partial X} + V \frac{\partial V}{\partial Y} = -\frac{\partial P}{\partial Y} + \frac{\nu_f}{\nu_{nf}} \frac{1}{Re} \nabla^2 V + \frac{Ri}{Pr} \frac{\beta_f}{\beta_{nf}} \Delta \theta \cos(\gamma) \tag{9}$$

and

$$U \frac{\partial \theta}{\partial X} + V \frac{\partial \theta}{\partial Y} = \frac{\alpha_f}{\alpha_{nf}} \nabla^2 \theta \tag{10}$$

2.1. Thermal Diffusivity And Effective Density

Thermal diffusivity and effective density of the nanofluid are

$$\alpha_{nf} = \frac{k_{nf}}{(\rho c_p)_{nf}} \tag{11}$$

$$\rho_{nf} = \phi \rho_s + (1 - \phi) \rho_f \tag{12}$$

2.2. Heat Capacity And Thermal Expansion Coefficient

Heat capacity and thermal expansion coefficient of the nanofluid are therefore

$$(\rho c_p)_{nf} = \phi (\rho c_p)_s + (1 - \phi) (\rho c_p)_f \tag{13}$$

$$(\rho\beta)_{nf} = \phi (\rho\beta)_s + (1 - \phi) (\rho\beta)_f \tag{14}$$

2.3. Viscosity

The effective viscosity of nanofluid was calculated by:

$$\mu_{eff} = \mu_f (1 + 2.5\phi) \left[1 + \eta \left(\frac{d_p}{L} \right)^{-2.6} \phi^{2/3} (\epsilon + 1) \right] \tag{15}$$

This well-validated model is presented by [Jang et al., \(2007\)](#) for a fluid containing a dilute

suspension of small rigid spherical particles and it accounts for the slip mechanism in nanofluids. The empirical constant ϵ and η are -0.25 and 280 for Al_2O_3 , respectively.

It is worth mentioning that the viscosity of the base fluid (water) is considered to vary with temperature and the flowing equation is used to evaluate the viscosity of water,

$$\mu_{H_2O} = (1.2723 \times T_{rc}^5 - 8.736 \times T_{rc}^4 + 33.708 \times T_{rc}^3 - 246.6 \times T_{rc}^2 + 518.78 \times T_{rc} + 1153.9) \times 10^6 \quad (16)$$

Where, $T_{rc} = \text{Log}(T - 273)$.

2.4. Dimensionless Stagnant Thermal Conductivity

The effective thermal conductivity of the nanoparticles in the liquid as stationary is calculated by the Hamilton and Crosser (H-C model) (Hamilton and Crosser, 1962) which is:

$$\frac{k_{stationary}}{k_f} = \frac{k_s + 2k_f - 2\phi(k_f - k_s)}{k_s + 2k_f + \phi(k_f - k_s)} \quad (17)$$

2.5. Total Dimensionless Thermal Conductivity Of Nanofluids

$$\frac{k_{nf}}{k_f} = \frac{k_{stationary}}{k_f} + \frac{k_c}{k_f} = \frac{k_s + 2k_f - 2\phi(k_f - k_s)}{k_s + 2k_f + \phi(k_f - k_s)} + c \frac{Nu_p d_f (2 - D_f) D_f \left[\left(\frac{d_{max}}{d_{min}} \right)^{1-D_f} - 1 \right]^2}{Pr(1 - D_f)^2 \left(\frac{d_{max}}{d_{min}} \right)^{2-D_f} - 1} \quad (18)$$

This model was proposed by Xu et al. (2006) and it has been chosen in this study to describe the thermal conductivity of nanofluids. c is an empirical constant (e.g. $c = 85$ for the deionized water and $c = 280$ for ethylene glycol) but independent of the type of nanoparticles. Nu_p is the Nusselt number for liquid flowing around a spherical particle and equal to two for a single particle in this work. The fluid molecular diameter $d_f = 4.5 \times 10^{-10}$ (m) for water in present study. The fractal dimension D_f is determined by:

$$D_f = 2 - \frac{\ln \phi}{\ln \left(\frac{d_{p,max}}{d_{p,min}} \right)}$$

Where, $d_{p,max}$ and $d_{p,min}$ are the maximum and minimum diameters of nanoparticles, respectively. Ratio of minimum to maximum nanoparticles $d_{p,min}/d_{p,max}$ is R .

$$d_{p,max} = d_p \cdot \frac{D_f - 1}{D_f} \left(\frac{d_{p,min}}{d_{p,max}} \right)^{-1}$$

$$d_{p,min} = d_p \cdot \frac{D_f - 1}{D_f}$$

NUMERICAL PROCEDURE

Governing equations for continuity, momentum and energy equations associated with the boundary conditions in this investigation were calculated numerically based on the finite volume method and associated staggered grid system, using FORTRAN computer code. To verify grid independence, numerical procedure was carried out for 9 different mesh sizes. Average Nu of the hot wall for 2 different situations are obtained for each grid size as shown in Fig 2.

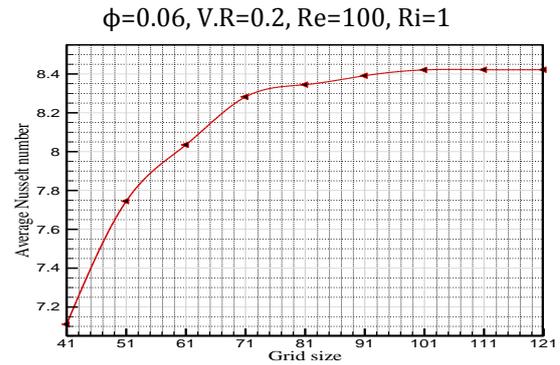


Figure 2: Mesh valid

RESULTS AND DISCUSSION

In this paper, flow pattern and thermal behavior of a lid-driven rectangular shallow cavity are investigated. The cavity has verified in two cases: one filled with six percent volume fraction of nanofluid and another filled with water as a base fluid. The cavity has adiabatic side moving walls with different velocities. In the present study, the length of cavity is twice of its width and the effect of velocity ratio of moving walls and existence of nanoparticles on hydrodynamic properties and thermal characteristics is discussed.

A variation in behavior of water and nanofluid with the variations of velocity ratio is presented in Figure 3. The comparison of fluid behavior in different velocity ratios in this figure and other figures declare the effect of variations in moving walls velocity ratio. Figure 3 shows flow pattern and thermal behavior of nanofluid and base fluid in different velocity ratios in $Ri=0.1$, $Re=100$, $A.R=2$. Flow pattern shows the formation of two vortices with different strength and intensity in two sides of cavity. The force due to the motion of walls on the fluid adjacent to the wall is the main reason of formation of vortices in this range of parameters. So the size and strength of vortices is a function of the velocity of the lateral walls. In this case and due to completely overcoming of shear force of walls on buoyancy force and alignment of these forces, only two

vortices are formed inside the cavity. Vortex induced by left wall motion is much stronger than other vortices in velocity ratios less than one and with increasing of velocity ratio; the vortex close to right wall is strengthened. Isotherm lines show convection heat transfer in all over the cavity. Density of lines formed in the vicinity of the upper and lower walls slightly increased with the increase of velocity ratio and consequently temperature gradient increases. On the other hand in this value of the Richardson number, ratio of natural convection to force convection is very little and dominant heat transfer mechanism in this case is force convection. Thus, due to absence of intermediate vortex induced by buoyancy force in this case, flow velocity in the middle compartment of cavity is low and it result in decreasing of overall heat transfer within cavity in lower Richardson numbers.

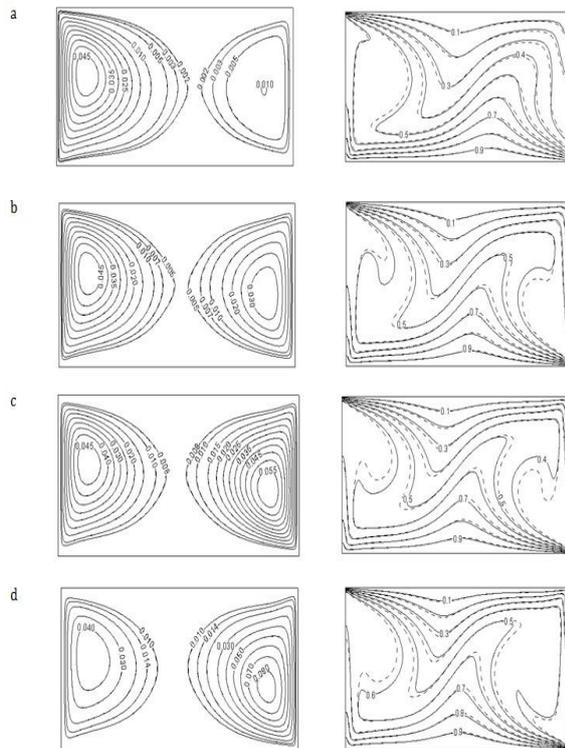


Figure 3: Streamlines and isotherms for base fluid (solid line) and nanofluid (dash line) at $Ri=0.1$, $Re=100$, $A.R=2$ a) $V.R=0.2$ b) $V.R=0.7$ c) $V.R=1$ d) $V.R=2$

Flow patterns and thermal behavior of base fluid and nanofluid in different velocity ratios and with $A.R=2$, $Ri=1$, $Re=100$ is shown in Figure 4. In the section a, increase of Richardson number relative to Figure 4 has created a central vortex in cavity and made flow pattern quite different, however, the strength and intensity of the double vortex is not too impressed. Thus, the

flow pattern in this case is composed of three vortices with different strengths that have covered almost all over the cavity.

Moreover the status of isotherm lines near the isotherm walls shows that increasing of velocity ratio and increasing of strength of right vortex result in increasing of density of isotherm lines in vicinity of these walls and consequently temperature gradient increases. With increasing of temperature gradient for nanofluid and according to convection heat transfer within cavity in this range of parameters, it is expected to make a huge increase in nanofluid heat transfer with increasing of velocity ratio. Exact analysis of this situation requires investigating of Nusselt diagrams.

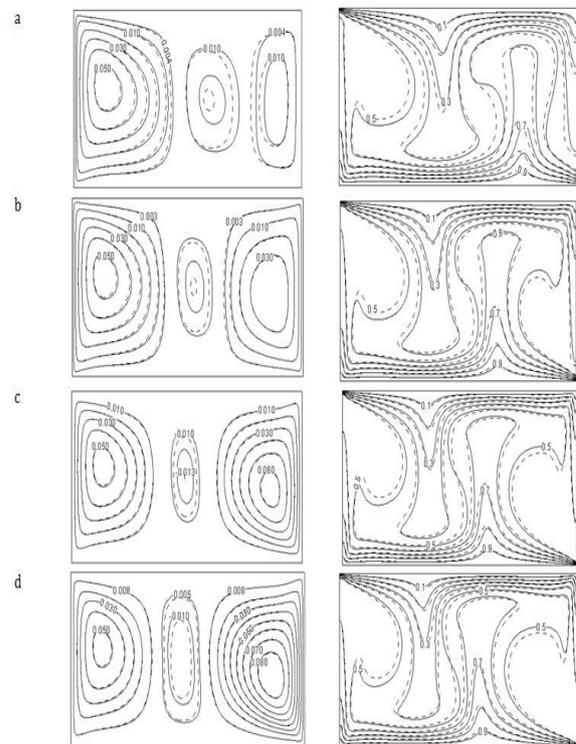


Figure 4: Streamlines and isotherms for base fluid (solid line) and nanofluid with solid volume fraction of 0.06 (dash line) at $Ri=1$, $Re=100$, $A.R=2$ a) $V.R=0.2$ b) $V.R=0.7$ c) $V.R=1$ d) $V.R=2$

CONCLUSIONS

In this paper, flow pattern and thermal behavior of a lid-driven rectangular shallow cavity are investigated. Effect of velocity ratio and existence of nanoparticles in base fluid are studied. Graphical and tabular results for various parametric conditions were presented and discussed. From this investigation, we can write the following conclusions:

1. By increasing of velocity ratio in all cases (except at very low values of the Reynolds number), the vortex near right wall develops

and rate of heat transfer increases. This increasing of rate of heat transfer depends on values of Reynolds number and Richardson number.

2. With an increase in Richardson number in a constant velocity ratio, Nusselt number and consequently rate of heat transfer also increase sharply.

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