

**SIMULATION OF COMBINED CONVECTION IN INCLINED TWO-SIDED LID DRIVEN ENCLOSURE
SUBJECTED TO NANOFLUID WITH DIFFERENT LIDS VELOCITY RATIO AND RICHARDSON NUMBER**

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ABSTRACT: The flow behaviour of Al_2O_3 nanofluids is investigated numerically inside a two-sided lid-driven inclined square cavity to gain insight into convective recirculation and flow processes induced by a nanofluid. The transport equations are solved numerically with finite volume approach using SIMPLE algorithm. Using the developed code, the effects of the Richardson number, the velocity ration of cavity lids and the volume fraction of the Cu nanoparticles on the flow inside the cavity are investigated. Governing parameters were $0.01 < Ri < 100$ but due to space constraints only the results for $0.1 < Ri < 10$ are presented.

KEYWORDS: Streamlines, Fluid Flow, Nanofluid, Mixed Convection.

INTRODUCTION

Mixed convection fluid flow and heat transfer are encountered in a number of engineering and industrial applications such as cooling of electronic equipment, chemical processing equipment, float glass manufacturing, food processing, and lubrication technologies.

There are a number of early studies on the mixed convection flow and heat transfer inside enclosures. Investigation of nanofluid behavior on free convection heat transfer can be found in a large number of articles. Free convection of nanofluids in simple rectangular enclosures with different boundary conditions has been studied by many authors.

[Khanafer et al. \(2003\)](#) were the first investigators who conduct a numerical study on free convection heat transfer inside rectangular cavities filled with nanofluids. They found that increase in the solid volume fraction of the nanofluid increases the rate of heat transfer for the entire range of Grashof number considered. [Santra et al. \(2008\)](#) studied free convection of Cu-water nanofluid in a differentially heated square cavity with the assumption of OstwaldedeWaele non-newtonian behavior of the nanofluid. They found that the heat transfer decreases when the solid volume fraction of the nanofluid increased for a particular Ra, while it increases with Ra for a particular solid volume fraction of the nanofluid. [Oztop and Abu-nada \(2008\)](#) carried out a numerical study on free convection of nanofluid in partially heated rectangular cavities. The cavities had a cold vertical wall, a localized heater on the other

vertical wall and insulated horizontal walls. They investigated effects of Rayleigh number, aspect ratio of cavities, size and location of the heater and different types of water based nanofluids.

The reported results in their study showed that average Nusselt number increased with increase in solid volume fraction of the nanofluid and increase in size of heater at all range of Rayleigh numbers considered. In another numerical study, [Abu-Nada and Oztop \(2009\)](#) investigated effect of inclination angle of a square cavity on free convection of the Cu;water nanofluid inside it. They observed that the inclination angle can be used as a control parameter for fluid flow and heat transfer. Moreover their results showed that effects of inclination angle on percentage of heat transfer enhancement become insignificant at low Rayleigh number.

There are a number of numerical studies about effects of nanofluid on combined natural and forced convection heat transfer (mixed convection heat transfer) in cavities.

[Tiwari and Das \(2007\)](#) studied numerically the mixed convection in two-sided lid-driven differentially heated square cavity filled with nanofluid. They showed that the additions of nanoparticles in a fluid are capable of increasing the heat transfer capacity of base fluid. As solid volume fraction increases, the effect is more pronounced. [Sebdani et al. \(2012\)](#) numerically studied the effects of Al_2O_3 /water nanofluid on mixed convection heat transfer in a square cavity with a heat source on the bottom wall and moving downward cold side walls. They prove that when the Reynolds number increases, while

the Rayleigh number is keeping constant, the forced convection becomes stronger that causes the heat transfer rate to increase. When the Rayleigh number increases, while the Reynolds number is kept constant, the heat transfer rate increases. The presence of nanoparticles causes increase in heat transfer rate only at $Ra = 10^3$ while at $Ra = 10^4$ and 10^5 the rate of heat transfer decreases with increase in nanoparticles volume fraction.

[Guo and Sharif, \(2004\)](#) used the finite volume method (FVM) and the SIMPLER algorithm to investigate the Mixed convection in rectangular cavities at different aspect ratios with moving isothermal sidewalls and constant heat flux source on the bottom wall. They studied the impact of the heat source length, the Richardson number and the aspect ratio of the cavity on the heat transfer. Their results showed that the average Nusselt number increased by moving the heat source towards the sidewalls. The problem of the available classical models is their inability to evaluate the effective viscosity and thermal conductivity of the nanofluids ([Morshed et al., 2008](#)). [Ho et al., \(2008\)](#) distinguished four recent models for the effective dynamic viscosity and thermal conductivity of alumina-water nanofluid on the natural convection in a square cavity. They concluded that the model used for the viscosity and the thermal conductivity of the nanofluid, plays as an important factor to predict that the heat transfer inside the enclosure could be either increased or decreased with respect to that of the base fluid. The Investigation of mixed convection flow and heat transfer in a cavity subjected to a Cu-water nanofluid has been executed by [Talebi et al., \(2010\)](#) using the finite volume method. The top and bottom walls of cavity were adiabatic and its vertical walls were differentially heated. They reported that for a specific Reynolds number, with an increase in the volume fraction of nanoparticles, heat transfer inside the cavity enhances. [Abu-Nada et al., \(2010\)](#) investigated the effects of variable properties on natural convection in cavities subjected to Al_2O_3 -water and CuO-water nanofluids. Their results indicated that the average Nusselt number is more affected by viscosity models than by thermal conductivity models at high Rayleigh numbers.

[Sharif, \(2007\)](#) examined laminar mixed convective heat transfer in a two-dimensional shallow inclined rectangular cavity. He found that average Nusselt number increases slowly with cavity inclination for the forced convection dominated case while it increases more rapidly with inclination angle for natural convection dominated case.

[Al-Amiri et al., \(2007\)](#) performed the effects of mixed convection heat transfer in lid-driven cavity with sinusoidal wavy bottom surface. They observed that the average Nusselt number increases as an increase in both amplitude of the wavy surface and Reynolds number.

[Basak et al., \(2009\)](#) investigated the influence of uniform and non-uniform heating of bottom wall on mixed convection lid-driven flows in a square cavity. They observed that heat transfer rate is very high at the edges of the bottom wall and it decreases at the center for the uniform heating. But there is lower heat transfer rate at edges for the non-uniform heating of the bottom wall.

[Roy et al., \(2004\)](#) has considered the case of radial flow cooling system for numerical simulation. For physical properties calculation they obtained a correlations by curve fitting on the experimental data and found that nanofluids increased the wall shear stress in a considerable way.

[Maiga et al., \(2004\)](#) have numerically simulated nanofluid behaviour in a uniformly heated tube for laminar as well as turbulent flow using approximated correlations for experimental data. They found that for turbulent flow regime, the heat transfer enhancement due to nanoparticles becomes more important with the increase of the Reynolds number.

In another study, [Maiga et al., \(2005\)](#) numerically simulated nanofluids in forced convection flows and solved the problems of uniformly heated tube and a system of parallel, coaxial and heated disks. They found that both the through flow Reynolds number and the gap between disks have insignificant effect on the heat transfer enhancement of nanofluids.

PHYSICAL MODELING AND GOVERNING EQUATIONS

A schematic view of the Two sided lid-driven cavity considered in the present study is shown in Figure 1. As shown in the figure, the lower one third segments of the left wall remains at high temperature T_h , however other portions of wall are adiabatic. The top and bottom walls of the cavity are assumed to move in their own plane with a constant velocity U_0 and U_0 multiplied by the velocity ratio ($U_0 \times V.R$), respectively.

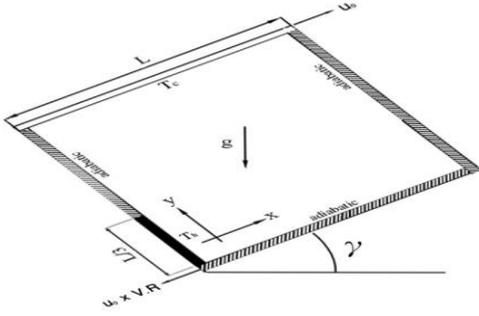


Figure 1: Schematic diagram of current study

The length of the cavity perpendicular to its plane is assumed to be long enough; hence, the problem is considered two-dimensional. The cavity is filled with Al_2O_3 water nanofluid. The thermophysical properties of Al_2O_3 nanoparticles and water as base fluid are listed in Table 1. The nanofluid is considered Newtonian, and the nanofluid flow is assumed to be laminar and incompressible. It is assumed that the nanoparticles and the base fluid are in thermal equilibrium and there is no slip between them. The density is varied according to the Boussinesq approximation.

Table 1: Thermo physical properties of base fluid and nanoparticles

Physical properties	Fluid phase (Water)	Solid (Al_2O_3)
C_p (J/kg k)	4179	765
ρ (kg/m ³)	21.	3970
K (W m ⁻¹ K ⁻¹)	0.6	25
$\beta \times 10^{-5}$ (1/K)		0.85

The continuity, momentum and energy equations which govern two-dimensional laminar mixed convection with the Boussinesq approximation in y-direction are as follows:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \quad (1)$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = -\frac{1}{\rho_{nf}} \frac{\partial p}{\partial x} + \nu_{nf} \nabla^2 u + \frac{(\rho\beta)_{nf}}{\rho_{nf}} g \Delta T \sin(\gamma), \quad (2)$$

$$u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} = -\frac{1}{\rho_{nf}} \frac{\partial p}{\partial y} + \nu_{nf} \nabla^2 v + \frac{(\rho\beta)_{nf}}{\rho_{nf}} g \Delta T \cos(\gamma) \quad (3)$$

And

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha_{nf} \nabla^2 T. \quad (4)$$

The dimensionless parameters may be presented as

$$X = \frac{x}{L}, \quad Y = \frac{y}{L}, \quad V = \frac{v}{U_0}, \quad U = \frac{u}{U_0} \quad (5)$$

$$\Delta T = T_h - T_c, \quad \theta = \frac{T - T_c}{\Delta T}, \quad P = \frac{p}{\rho_{nf} U_0^2}.$$

Hence,

$$Re = \frac{\rho_f U_0 L}{\mu_f}, \quad Ri = \frac{Ra}{Pr \cdot Re^2}, \quad (6)$$

$$Ra = \frac{g B_f \Delta T L^3}{\nu_f \alpha_f}, \quad Pr = \frac{\nu_f}{\alpha_f}.$$

The dimensionless forms of the above governing equations (1) to (4) become:

$$\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0 \quad (7)$$

$$U \frac{\partial U}{\partial X} + V \frac{\partial U}{\partial Y} = -\frac{\partial P}{\partial X} + \frac{\nu_{nf}}{\nu_f} \frac{1}{Re} \nabla^2 U + \frac{Ri}{Pr} \frac{\beta_{nf}}{\beta_f} \Delta \theta \sin(\gamma) \quad (8)$$

$$U \frac{\partial V}{\partial X} + V \frac{\partial V}{\partial Y} = -\frac{\partial P}{\partial Y} + \frac{\nu_{nf}}{\nu_f} \frac{1}{Re} \nabla^2 V + \frac{Ri}{Pr} \frac{\beta_{nf}}{\beta_f} \Delta \theta \cos(\gamma) \quad (9)$$

And

$$U \frac{\partial \theta}{\partial X} + V \frac{\partial \theta}{\partial Y} = \frac{\alpha_{nf}}{\alpha_f} \nabla^2 \theta \quad (10)$$

Thermal diffusivity and effective density of the nanofluid are:

$$\alpha_{nf} = \frac{k_{nf}}{(\rho c_p)_{nf}} \quad (11)$$

$$\rho_{nf} = \phi \rho_s + (1 - \phi) \rho_f \quad (12)$$

Heat capacity and thermal expansion coefficient of the nanofluid are therefore:

$$(\rho c_p)_{nf} = \phi (\rho c_p)_s + (1 - \phi) (\rho c_p)_f \quad (13)$$

$$(\rho \beta)_{nf} = \phi (\rho \beta)_s + (1 - \phi) (\rho \beta)_f \quad (14)$$

The effective viscosity of nanofluid was proposed by Brinkman (1952). as below:

$$\mu_{nf} = \frac{\mu_f}{(1 - \phi)^{2.5}} \quad (15)$$

The effective thermal conductivity of the nanofluid is calculated by the Maxwell model (1904) which is:

$$\frac{k_{nf}}{k_f} = \frac{k_s + 2k_f - 2\phi(k_f - k_s)}{k_s + 2k_f + \phi(k_f - k_s)} \quad (16)$$

The Nusselt number can be calculated as:

$$Nu = \frac{hL}{k_f} \quad (17)$$

Where the heat transfer coefficient h is defined as:

$$h = \frac{q_w}{T_h - T_c} \quad (18)$$

And the thermal conductivity may be express as:

$$k_{nf} = \frac{-q_w}{\partial T / \partial X} \tag{19}$$

The average Nusselt number calculated over the hot surface by Eq. (18) becomes

$$Nu_m = \frac{1}{L_h} \int_0^{L_h} Nu dY \tag{21}$$

NUMERICAL PROCEDURE

The governing equations are discretized using the finite volume method. The coupling between the pressure and the velocity is done using the SIMPLER algorithm. The diffusion terms in the equations are discretized by a second order central difference scheme while

a hybrid scheme (a combination of the central difference scheme and the upwind scheme) is employed to approximate the convection terms. The set of discretized equations are solved by TDMA line by line method.

The solution procedure is repeated until the following convergence criterion is satisfied

$$error = \frac{\sum_{j=1}^{j=M} \sum_{i=1}^{i=N} |\lambda^{n+1} - \lambda^n|}{\sum_{j=1}^{j=M} \sum_{i=1}^{i=N} |\lambda^{n+1}|} < 10^{-7} \tag{23}$$

Here, M and N correspond to the number of grid points in x and y directions; respectively. n is the number of iteration and λ denotes any scalar transport quantity. To verify grid independence, numerical procedure was carried out for nine different mesh sizes, namely; 21 × 21, 31 × 31, 41 × 41, 51 × 51, 61 × 61, 71 × 71, 81 × 81, 91 ×

91 and 101 × 101. Average Nu of the right hot wall is obtained for each grid size as shown in Figure 2. As can be observed, an 81 × 81 uniform grid size yields the required accuracy and was hence applied for all simulation exercises in this work as presented in the following section.

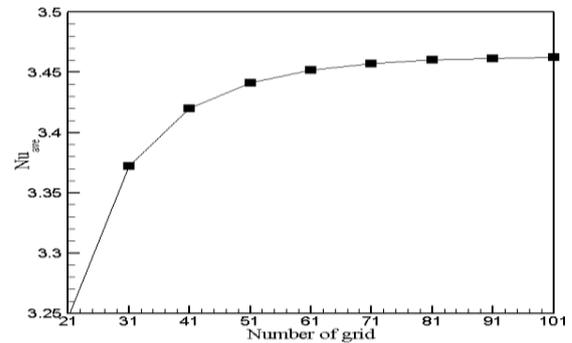


Figure 2: mesh grid validation

RESULTS AND DISCUSSION

In this paper, a numerical analysis has been conducted to investigate the effects of inclination angles and velocity ration on a nanofluid filling a two dimensional inclined double lid-driven cavity (Figure 3). There is a constant length of the heating portion along the hot wall. The remaining portions are adiabatic while partially heating the left wall. Mixed convection is a kind of convection including both natural and forced convection. A number of non-dimensional parameters are involved in the flow processes. Therefore, it is essential that the parametric studies should be carefully sorted out to cover the range of important parameters involved in the study.

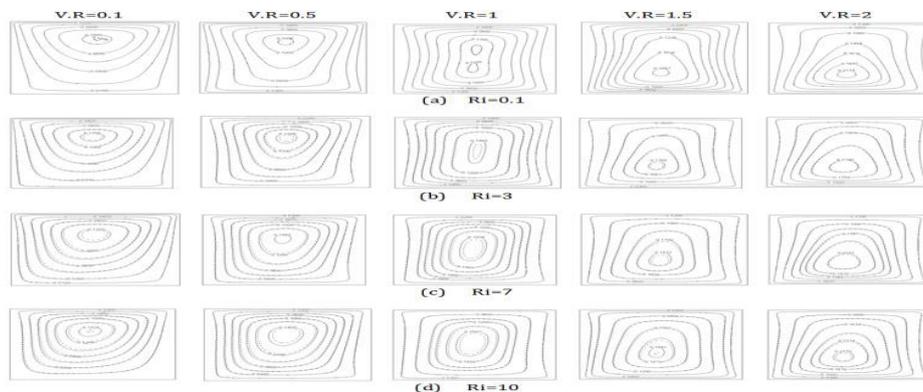


Figure 3: Streamlines for nanofluid with $\phi= 0.06$ (dashed line) and pure fluid (solid line) for different Richardson numbers and different velocity ratios (V. R.) with $Re=10$ and $\gamma=0$.

The Richardson number plays an important role to determine the mode of convection.

Richardson number is the measure of the relative importance of the buoyancy-driven

natural convection to the forced convection. based on the values of the Richardson number, the problem of the mixed shear and buoyancy-driven convection can be divided into three flow regimes as follows; pure natural convection for $Ri \gg 10$, pure forced convection for $Ri \ll 1$, and mixed convection for $0.1 < Ri < 10$. It is observed that the shear force is more dominant than the buoyancy in the mixed convection region.

Fluid flow and heat distribution are depicted in Figure 3 in the form of streamlines and isotherms, respectively. The velocity ratio of moving walls is changed from 0.1 to 2 as Richardson number is increased from 0.1 to 10.

Figure 3(a)–(d) exhibits streamlines for different velocity ratios and the Richardson numbers with $Re=10$ and $\gamma=0$. The fluid flows for $Ri=0.1$, while the velocity ratio is increasing, exists in Figure 3(a). The flow pattern containing a single major cell for $V.R=0.1$. The streamlines are concentrated near the moving walls due to the shear forces. As the velocity ratio is increased to 0.5, the major cell is gradually moving down. It is observed that the single major cell splits into two symmetric cells for $V.R=1$. As the velocity ratio is increased to 0.5. With increasing of $V.R$ from 1 to 1.5, two separated cells stick together and create a major cell. This cell moves down, with increasing of velocity ratio from 1.5 to 2.

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