

## NUMERICAL MODELING OF NATURAL AND FORCED CONVECTIVE FLUID FLOW INSIDE AN ENCLOSURE WITH USING FINITE VOLUME METHOD

Tavoosi M, kazemi M, Shirmohammadi N, Khosravifard M, Hashemi M  
*Department of Mechanical Engineering, Najafabad Branch, Islamic Azad University, Isfahan, Iran*

**ABSTRACT:** In this paper, natural and forced convection heat transfer in a square enclosure is investigated. The right and left walls of the mentioned cavity are insulated while the top and bottom wall is exposed to a constant temperature. Fluid flow for Silicon Oxide nanoparticles for Reynolds number between 1 to 300, Richardson number between 0.1 to 1, solid volume fraction between 0 to 0.06 and inclination angles between 0° to 150° are obtained and discussed.

**KEYWORDS:** Streamlines, Lid driven, Boussinesq approximation, Numerical simulation

### INTRODUCTION

Low thermal conductivity of conventional heat transfer fluids such as water, oil, and ethylene glycol mixture is a primary limitation in enhancing the performance and the compactness of many engineering electronic devices. To overcome this drawback, there is a strong motivation to develop advanced heat transfer fluids with substantially higher conductivities to enhance thermal characteristics. As such an innovative way in improving thermal conductivities of a fluid is to suspend metallic nanoparticles within it. The resulting mixture referred to as a nanofluid possesses a substantially larger thermal conductivity compared to that of the traditional fluids ([Eastman et al., 2001](#)). The presence of the nanoparticles in the fluids increases appreciably the effective thermal conductivity of the fluid and consequently enhances the heat transfer characteristics.

[Khanafer et al., \(2003\)](#) investigated the heat transfer enhancement in a two-dimensional enclosure utilizing nanofluids for a range of Grashof numbers and volume fractions. It was found that the heat transfer across the enclosure was found to increase with the volumetric fraction of the copper nanoparticles in water at any given Grashof number. [Kim et al., \(2007\)](#) studied the pool boiling characteristics of dilute dispersions of Al<sub>2</sub>O<sub>3</sub>, ZrO<sub>2</sub> and SiO<sub>2</sub> nanoparticles in water. It was found that a significant enhancement in critical heat flux can be obtained at the modest nanoparticle concentration. [Wen and Ding, \(2005\)](#) investigated the heat transfer enhancement using water-TiO<sub>2</sub> nanofluid filled in a

rectangular enclosure heated from below. They reported that for the Rayleigh number less than 106, the natural convection heat transfer rate increasingly decreased with the increase of particle concentration, particularly at low Rayleigh number. [Ho et al., \(2008\)](#) investigated the influences of uncertainties due to adopting various formulas for the effective thermal conductivity and dynamic viscosity of alumina/water nanofluid on the heat transfer characteristics. It was found that the uncertainties associated with different formulas adopted for the effective thermal conductivity and dynamic viscosity of the nanofluid have a strong bearing on the natural convection heat transfer characteristics in the enclosure.

So many investigators have experimentally studied flow and thermal characteristics of nanofluids. Especially, in order to understanding buoyancy-driven heat transfer of nanofluids in a cavity several investigations have been theoretically and experimentally conducted. [Putra et al., \(2003\)](#) conducted the experiment for observation on the natural convective characteristics of water based on Al<sub>2</sub>O<sub>3</sub>. They reported that natural convective heat transfer in a cavity is decreased with the increment of the volume fraction of nanoparticles.

[Kim et al., \(2004\)](#) analytically researched the convective instability driven by buoyancy and heat transfer characteristics of nanofluids with theoretical models which are used to estimate properties of nanofluids and indicated that as the thermal conductivity and shape factor of nanoparticles decrease, the convective motion in a nanofluid sets in easily and their results were similar with Putra et al. experimental

investigation. [Abu-Nada et al., \(2008\)](#) investigated the influences of nanoparticle on the natural convection heat transfer enhancement in horizontal annuli with various nanoparticles and volume fractions. They reported an enhancement of heat transfer in horizontal annuli.

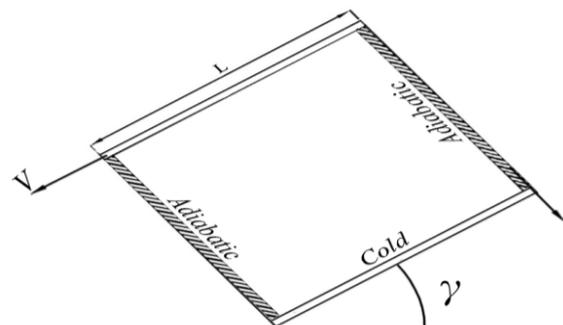
Natural convection in fluid-filled rectangular enclosures has received considerable attention over the past several years due to its wide applications in engineering design of advances technology. The first study concerning natural convection of a nanofluid confined in a differentially heated enclosure seems to be due to [Khanafar et al., \(2003\)](#). A comparative study of different models based on the thermophysical properties of copperwater nanofluid is developed and investigated. Their numerical results indicate that the suspended nanoparticles substantially increase the heat transfer rate at any given Grashof number. A heat transfer correlation of the average Nusselt number for various Grashof numbers and volume fraction is proposed by the authors. The same problem was considered by [Jou and Tzeng, \(2006\)](#). The Khanafar et al.'s model was used to investigate the convective heat transfer enhancement in rectangular enclosures filled with and Al<sub>2</sub>O<sub>3</sub>-water nanofluid. It was also reported that increasing the buoyancy parameter and volume fraction cause an increase in the average heat transfer coefficient. Natural convection heat transfer of nanofluids in a square cavity, heated isothermally from the vertical sides, has been investigated numerically by [Ho et al., \(2008\)](#) and [Santra et al., \(2008\)](#).

Mixed convection in a two-sided lid-driven differentially heated square cavity was studied by [Oztop and Dagtekin, \(2004\)](#). They determined that both Richardson number and direction of moving walls affect the fluid flow and heat transfer inside the cavity. [Al-Amiri et al., \(2007\)](#) analyzed steady mixed convection in a square lid-driven cavity under the combined buoyancy effects of thermal and mass diffusion. They investigated that heat and mass transfer characteristics inside the cavity are enhanced for low values of the Richardson number due to the dominant effect induced by moving lid. [Sharif, \(2007\)](#) examined laminar mixed convective heat transfer in a two-dimensional shallow inclined rectangular cavity. He found that average Nusselt number increases slowly with cavity inclination for the forced convection dominated case while it increases more rapidly with inclination angle for natural convection dominated case.

[Mohamad and Viskanta, \(1995\)](#) investigated the effects of a sliding lid on the fluid flow and thermal structures in a shallow lid-driven cavity. They found that the maximum local heat transfer rate occurs at the starting area of the sliding lid and decreases along the sliding lid. [Chenak et al., \(1995\)](#) studied mixed convection and conduction heat transfer in open cavities. They showed that heat transfer across the cavity is enhanced when the cavity aspect ratio is increased. [Prasad and Koseff, \(1996\)](#) investigated combined forced and natural convection in a lid-driven cavity experimentally. [Chamkha, \(2002\)](#) investigated the problem of unsteady laminar combined convection flow and heat transfer of an electrically conducting and heat generating or absorbing fluid in a vertical lid-driven cavity in the presence of a uniform magnetic field. He found that the presence of the internal heat generation effect is found to decrease the average Nusselt number significantly for aiding flow and to increase it for opposing flow.

#### PROBLEM STATEMENT

Fig. 1 shows a two-dimensional square cavity considered for the present study with physical dimensions. The height and the width of the square cavity are denoted  $L$ . The length of the cavity perpendicular to the plane of the geometry is assumed to be long enough; hence the problem is considered two-dimensional. The right and left walls are kept insulated whereas the top and bottom walls are maintained at constant temperatures.



**Figure 1:** A schematic view of the cavity considered in the present study.

The fluid used in this study is assumed to be Newtonian and incompressible and the fluid flow is assumed to be laminar. The base fluid (water) and the nanoparticles are in thermal equilibrium and there is no slip between them. The thermophysical properties of the nanofluid are assumed to be constant with the exception of density which varies according to the Boussinesq approximation.

The governing equations for a steady, two-dimensional laminar and incompressible flow are expressed as:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \quad (1)$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = -\frac{1}{\rho_{nf}} \frac{\partial p}{\partial x} + \nu_{nf} \nabla^2 u + \frac{(\rho\beta)_{nf}}{\rho_{nf}} g \Delta T \sin(\gamma), \quad (2)$$

$$u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} = -\frac{1}{\rho_{nf}} \frac{\partial p}{\partial y} + \nu_{nf} \nabla^2 v + \frac{(\rho\beta)_{nf}}{\rho_{nf}} g \Delta T \cos(\gamma) \quad (3)$$

And

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha_{nf} \nabla^2 T. \quad (4)$$

The dimensionless parameters may be presented as

$$X = \frac{x}{L}, \quad Y = \frac{y}{L}, \quad V = \frac{v}{U_0}, \quad U = \frac{u}{U_0} \quad (5)$$

$$\Delta T = T_h - T_c, \quad \theta = \frac{T - T_c}{\Delta T}, \quad P = \frac{p}{\rho_{nf} U_0^2}.$$

Hence,

$$Re = \frac{\rho_f U_0 L}{\mu_f}, \quad Ri = \frac{Ra}{Pr \cdot Re^2}, \quad (6)$$

$$Ra = \frac{g B_f \Delta T L^3}{\nu_f \alpha_f}, \quad Pr = \frac{\nu_f}{\alpha_f}.$$

The dimensionless forms of the above governing equations (1) to (4) become:

$$\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0 \quad (7)$$

$$U \frac{\partial U}{\partial X} + V \frac{\partial U}{\partial Y} = -\frac{\partial P}{\partial X} + \frac{\nu_{nf}}{\nu_f} \frac{1}{Re} \nabla^2 U + \frac{Ri}{Pr} \frac{\beta_{nf}}{\beta_f} \Delta \theta \sin(\gamma) \quad (8)$$

$$U \frac{\partial V}{\partial X} + V \frac{\partial V}{\partial Y} = -\frac{\partial P}{\partial Y} + \frac{\nu_{nf}}{\nu_f} \frac{1}{Re} \nabla^2 V + \frac{Ri}{Pr} \frac{\beta_{nf}}{\beta_f} \Delta \theta \cos(\gamma) \quad (9)$$

And

$$U \frac{\partial \theta}{\partial X} + V \frac{\partial \theta}{\partial Y} = \frac{\alpha_{nf}}{\alpha_f} \nabla^2 \theta \quad (10)$$

Thermal diffusivity and effective density of the nanofluid are:

$$\alpha_{nf} = \frac{k_{nf}}{(\rho c_p)_{nf}} \quad (11)$$

$$\rho_{nf} = \phi \rho_s + (1 - \phi) \rho_f \quad (12)$$

Heat capacity and thermal expansion coefficient of the nanofluid are therefore:

$$(\rho c_p)_{nf} = \phi (\rho c_p)_s + (1 - \phi) (\rho c_p)_f \quad (13)$$

$$(\rho \beta)_{nf} = \phi (\rho \beta)_s + (1 - \phi) (\rho \beta)_f \quad (14)$$

The effective viscosity of nanofluid was proposed by Brinkman (1952), as below:

$$\mu_{nf} = \frac{\mu_f}{(1 - \phi)^{2.5}} \quad (15)$$

The effective thermal conductivity of the nanofluid is calculated by the Maxwell model (1904) which is:

$$\frac{k_{nf}}{k_f} = \frac{k_s + 2k_f - 2\phi(k_f - k_s)}{k_s + 2k_f + \phi(k_f - k_s)} \quad (16)$$

The Nusselt number can be calculated as:

$$Nu = \frac{hL}{k_f} \quad (17)$$

Where the heat transfer coefficient h is defined as:

$$h = \frac{q_w}{T_h - T_c} \quad (18)$$

#### NUMERICAL IMPLEMENTATION

The governing equations are discretized using the finite volume method. The coupling between the pressure and the velocity is done using the SIMPLER algorithm. The diffusion terms in the equations are discretized by a second order central difference scheme while

a hybrid scheme (a combination of the central difference scheme and the upwind scheme) is employed to approximate the convection terms. The set of discretized equations are solved by TDMA line by line method. In order to validate the proposed numerical scheme, the free convection in a Cu-water filled square cavity with cold right wall, partially heated left wall and insulated horizontal walls is analyzed using the presented code, and the results are compared with the results of [Oztop and Dagtekin, \(2004\)](#) for the same problem. It is observed that very good agreements exist between the two results.

The solution procedure is repeated until the following convergence criterion is satisfied

$$\text{error} = \frac{\sum_{j=1}^{j=M} \sum_{i=1}^{i=N} |\lambda^{n+1} - \lambda^n|}{\sum_{j=1}^{j=M} \sum_{i=1}^{i=N} |\lambda^{n+1}|} < 10^{-7}. \quad (19)$$

Here, M and N denotes the number of grid points in x and y directions, respectively. N is the number of iteration and  $\lambda$  denotes any scalar transport quantity.

To verify grid independence, nine different grid sizes are tested from 21 × 21, 31 × 31, 41 × 41, 51 × 51, 61 × 61, 71 × 71, 81 × 81, 91 × 91 and 101 × 101. Average Nusselt number of the hot

wall is obtained for each grid size as shown in fig 2.

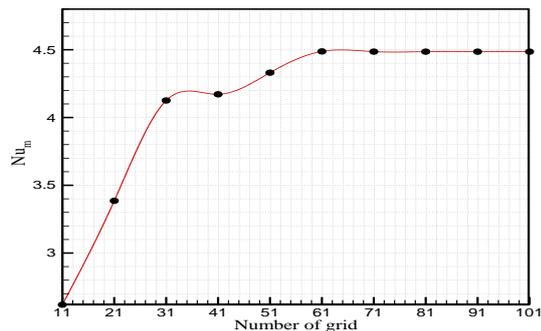


Figure 2: Mesh grid validation.

In this paper, natural and forced convection heat transfer in a square enclosure is investigated. The right and left walls of the mentioned cavity are insulated while the top and bottom wall is exposed to a constant temperature. Fluid flow for Silicon Oxide nanoparticles for Re between 1 to 300, Ri between 0.1 to 1,  $\phi$  between 0 to 0.06 and  $\gamma$  between 0 to 150 are obtained and discussed. These results include streamlines. Plots of the streamlines and the isotherm for Ri=1, and different Re and different  $\gamma$  for nanofluid ( $\phi=0.06$ ) and base fluid are shown in Figures (3).

RESULTS AND DISCUSSION

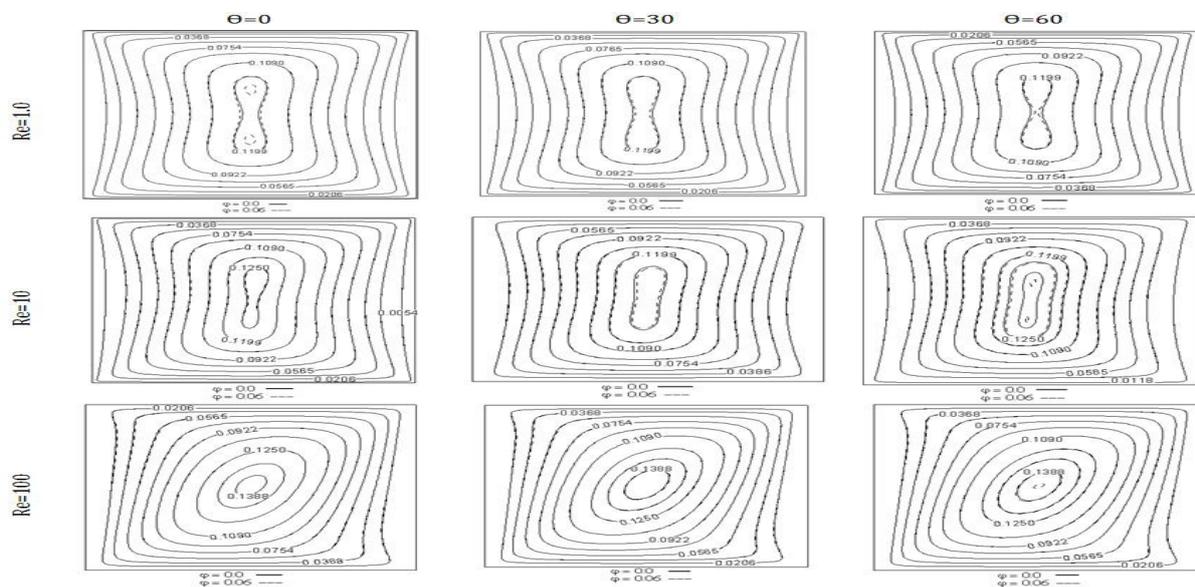


Figure 3: Streamlines for nanofluid.

As can be seen in these figures, by increasing Re in a specific  $\gamma$ , intensity of the streamlines becomes more. Therefore, mass flow rate between two specific point's increases, thus, this phenomenon leads to increasing the rate of heat transfer and the average Nusselt number. Increasing  $\phi$  does not have considerable effect on the streamlines. However in some figures related to Re= 100, the streamlines are becomes close to the wall. Therefore in the lower area of the heated wall at this slope, a small vortex is formed. By increasing  $\phi$ , the streamlines tend to get closer to the walls. So the amount of mass flow rate that is flowing near to the heat source increases thus increasing the rate of heat transfer.

At  $\gamma=0^\circ$  and for lower Reynolds number, due to slow movement of the upper and the lower walls, the streamlines are closer to these walls than to the left and right walls. Thus, forms a vertically stretched vortex in the center of the

cavity. In the upper and lower areas of this vortex, two small rotational flows are formed. By increasing Re number to 10 and then to 100, velocity of the horizontal walls increases which causes a horizontal stretch in the fluid. Therefore, the height of the central vortex decreases.

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