

## INVESTIGATION OF FLUID FLOW AND STREAMLINES IN DIFFERENT REYNOLDS NUMBER AND VARIOUS INCLINATION ANGLES

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**ABSTRACT:** In this article, combined convection heat transfer in a square cavity is studied. The left and right sides of cavity kept at constant temperature while horizontal sides are insulated. Fluid flow of  $Al_2O_3$  nanoparticles for Re between 1 to 300, Ri between 0.1 to 1, and different  $\phi$  and  $\gamma$  are calculated and discussed.

**KEYWORDS:** fluid flow, nanofluid, Reynolds number, Richardson number, inclination angles.

### INTRODUCTION

Fluids with suspended nanoparticles are called nanofluids, a term first proposed by Choi in 1995 of the Argonne National Laboratory, U.S.A. ([Choi et al., 2004](#)). Nanofluid is considered to be the next-generation heat transfer fluids as they offer exciting new possibilities to enhance heat transfer performance compared to pure liquids. Recently, researchers have demonstrated that nanofluids (such as water or ethylene glycol) with CuO or  $Al_2O_3$  nanoparticles exhibit enhanced thermal conductivity ([Choi et al., 2004](#)). Thus, the use of nanofluids, for example in heat exchangers, may result in energy and cost savings and should facilitate the trend of device miniaturization. Mixed convection in a lid-driven cavity has received much attention. In their investigation, [Choi et al., \(2004\)](#) investigated computationally the flow of a viscous thermally stratified fluid in a square cavity.

[Ho et al., \(2008\)](#) investigated influences of uncertainties due to adapting different models for the effective thermal conductivity and the dynamic viscosity of alumina-water nanofluid on the natural convection heat transfer in a square cavity.

Their results showed that the heat transfer across the cavity could be either enhanced or mitigated with respect to that of the base fluid depending on the model used for the thermal conductivity and the viscosity of the nanofluid. Using the control volume method, [Muthtamilselvan et al., \(2010\)](#) investigated the mixed convection heat transfer in a lid-driven rectangular enclosure filled with the Copperwater nanofluid. The enclosure's side walls were

insulated while its horizontal walls were kept at constant temperatures, with the top wall moving at a constant velocity. They observed that both the aspect ratio of the cavity as well as the nanoparticles volume fraction affected the fluid flow and heat transfer inside the enclosure. Mixed convection in enclosures for various different boundary conditions has been studied by [Gebhart et al., \(1988\)](#) and [Hasanoui et al., \(1990\)](#). [Oztop and Varol, \(2009\)](#) performed a numerical study to obtain combined convection field in inclined porous lid-driven enclosures heated from one wall with a non-uniformly heater. It was observed that flow field, temperature distribution and heat transfer are affected by inclination angle of the enclosure. [Lee and Chen, \(1996\)](#) obtained finite element solutions of mixed convection in a bottom heated square cavity. [Moraga and Lopez, \(2004\)](#) performed a numerical analysis of three-dimensional model of mixed convection in an air-cooled cavity in order to compare the variations in different properties with the results of two-dimensional models. [Wang and Chen, \(2009\)](#) analyzed forced convection in a wavy-wall channel and demonstrated the effects of wavy geometry, Reynolds number and Prandtl number on the skin friction and Nusselt number.

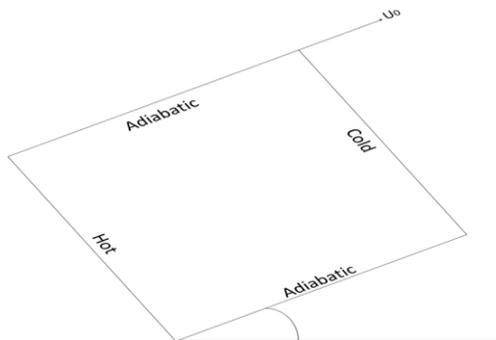
A comparative study of different models based on the thermophysical properties of copper-water nanofluid is developed and investigated. Their numerical results indicate that the suspended nanoparticles substantially increase the heat transfer rate at any given Grashof number. A heat transfer correlation of the average Nusselt

number for various Grashof numbers and volume fraction is proposed by the authors. The same problem was considered by [Jou and Tzeng, \(2006\)](#). The Khanafer's model was used to investigate the convective heat transfer enhancement in rectangular enclosures filled with an Al<sub>2</sub>O<sub>3</sub>-water nanofluid. It was also reported that increasing the buoyancy parameter and volume fraction cause an increase in the average heat transfer coefficient. Natural convection heat transfer of nanofluids in a square cavity, heated isothermally from the vertical sides, has been investigated numerically by [Ho et al., \(2008\)](#) and [Santra et al., \(2008\)](#). [Chen et al., \(2007\)](#) considered Darcy–Brinkman–Forchheimer extended model to examine free convection inside a porous cavity. This model has been initially introduced by [Brinkman, \(1947\)](#) in order to account for the transition from Darcy flow to highly viscous flow, in the limit of high permeability. Darcy–Forchheimer model is based on the effect of inertia and viscous forces in the porous media.

As mentioned, augmentation of nano-particles in the base fluid makes a higher thermal conductivity nanofluid. However, the shape, size, and thermal properties of the solid particles in nanofluids have important roles in the thermal conductivity ([Eastman et al., 2001](#)), but the role of high contact surface of nanoparticles in enhancement of heat transfer abilities more significant than the role of larger particles ([Xuan and Li, 2000](#)).

### MODELING AND GOVERNING EQUATIONS

Figure 1 shows a schematic diagram of square cavity considered for the present study with physical dimensions. The cavity is filled with Al<sub>2</sub>O<sub>3</sub>-water nanofluid and having adiabatic upper moving lid. Right and left walls are kept in constant temperature while the bottom wall is assumed to be insulated. The length of the cavity perpendicular to its plane is assumed to be long enough; hence, the study is considered two dimensional.



**Figure 1:** Schematic diagram of current study

The cavity is subjected to a suspension of Al<sub>2</sub>O<sub>3</sub> nanoparticles in water that there is no slip between them. The nanofluid is assumed to be incompressible, and nanoparticles and the base fluid are in thermal equilibrium. The thermophysical properties of nanoparticles and the water as the base fluid at T = 25°C are presented in Table1.

**Table 1:** thermophysical properties of water and Al2O3

Physical properties	Fluid phase (Water)	Solid (Al <sub>2</sub> O <sub>3</sub> )
Cp (J/kg k)	4179	765
ρ (kg/m <sup>3</sup> )	997.1	3970
K (W m <sup>-1</sup> K <sup>-1</sup> )	0.6	25
β × 10 <sup>-5</sup> (1/K)	21.	0.85
μ × 10 <sup>-4</sup> (Kg/ms)	8.9	.....

The governing equations for a steady, two-dimensional laminar and incompressible flow are expressed as:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \quad (1)$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = -\frac{1}{\rho_{nf}} \frac{\partial p}{\partial x} + \nu_{nf} \nabla^2 u, \quad (2)$$

$$u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} = -\frac{1}{\rho_{nf}} \frac{\partial p}{\partial y} + \nu_{nf} \nabla^2 v + \frac{(\rho\beta)_{nf}}{\rho_{nf}} g \Delta T, \quad (3)$$

And

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha_{nf} \nabla^2 T. \quad (4)$$

The dimensionless parameters may be presented as

$$X = \frac{x}{L}, \quad Y = \frac{y}{L}, \quad V = \frac{v}{u_0}, \quad U = \frac{u}{u_0} \quad (5)$$

$$\Delta T = T_h - T_c, \quad \theta = \frac{T - T_c}{\Delta T}, \quad P = \frac{p}{\rho_{nf} u_0^2}.$$

Hence,

$$Re = \frac{\rho_f u_0 L}{\mu_f}, \quad Ri = \frac{Ra}{Pr \cdot Re^2}, \quad Ra = \frac{g \beta_f \Delta T L^3}{\nu_f \alpha_f}, \quad Pr = \frac{\nu_f}{\alpha_f}. \quad (6)$$

The dimensionless form of the above governing equations (1) to (4) become

$$\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0 \quad (7)$$

$$U \frac{\partial U}{\partial X} + V \frac{\partial U}{\partial Y} = -\frac{\partial P}{\partial X} + \frac{\nu_{nf}}{\nu_f} \frac{1}{Re} \nabla^2 U \quad (8)$$

$$U \frac{\partial V}{\partial X} + V \frac{\partial V}{\partial Y} = -\frac{\partial P}{\partial Y} + \frac{\nu_{nf}}{\nu_f} \frac{1}{Re} \nabla^2 V + \frac{Ri}{Pr} \frac{\beta_{nf}}{\beta_f} \Delta \theta \quad (9)$$

And

$$U \frac{\partial \theta}{\partial X} + V \frac{\partial \theta}{\partial Y} = \frac{\alpha_{nf}}{\alpha_f} \nabla^2 \theta \quad (10)$$

### 2.1. Thermal diffusivity and effective density

Thermal diffusivity and effective density of the nanofluid are

$$\alpha_{nf} = \frac{k_{nf}}{(\rho c_p)_{nf}} \quad (11)$$

$$\rho_{nf} = \varphi \rho_s + (1 - \varphi) \rho_f \quad (12)$$

### 2.2. Heat capacity and thermal expansion coefficient

Heat capacity and thermal expansion coefficient of the nanofluid are therefore

$$(\rho c_p)_{nf} = \varphi (\rho c_p)_s + (1 - \varphi) (\rho c_p)_f \quad (13)$$

$$(\rho \beta)_{nf} = \varphi (\rho \beta)_s + (1 - \varphi) (\rho \beta)_f \quad (14)$$

### 2.3. Dynamic viscosity of nanofluid

The effective viscosity of nanofluid was calculated by:

$$\mu_{eff} = \mu_f (1 + 2.5\varphi) \left[ 1 + \eta \left( \frac{d_p}{L} \right)^{-2\varepsilon} \varphi^{2/3} (\varepsilon + 1) \right] \quad (15)$$

This well-validated model is presented by [Jang et al. \(2007\)](#) for a fluid containing a dilute suspension of small rigid spherical particles and it accounts for the slip mechanism in nanofluids. The empirical constant  $\varepsilon$  and  $\eta$  are -0.25 and 280 for  $Al_2O_3$ , respectively.

It is worth mentioning that the viscosity of the base fluid (water) is considered to vary with temperature and the flowing equation is used to evaluate the viscosity of water,

$$\mu_{H_2O} = (1.2723 \times T_{rc}^{-5} - 8.736 \times T_{rc}^{-4} + 33.708 \times T_{rc}^{-3} - 246.6 \times T_{rc}^{-2} + 518.78 \times T_{rc} + 1153.9) \times 10^6 \quad (16)$$

Where,

$$T_{rc} = \text{Log}(T - 273).$$

### 2.4. Dimensionless stagnant thermal conductivity

The effective thermal conductivity of the nanoparticles in the liquid as stationary is calculated by the Hamilton and crosser (H-C model; [Hamilton and Crosser, 1962](#)), which is:

$$\frac{k_{stationary}}{k_f} = \frac{k_s + 2k_f - 2\varphi(k_f - k_s)}{k_s + 2k_f + \varphi(k_f - k_s)} \quad (17)$$

## NUMERICAL IMPLEMENTATION

The governing equations are discretized using the finite volume method. The coupling between the pressure and the velocity is done using the SIMPLER algorithm. The diffusion terms in the

equations are discretized by a second order central difference scheme while a hybrid scheme (a combination of the central difference scheme and the upwind scheme) is employed to approximate the convection terms. The set of discretized equations are solved by TDMA line by line method. In order to validate the proposed numerical scheme, the free convection in a Cu-water filled square cavity with cold right wall, partially heated left wall and insulated horizontal walls is analyzed using the presented code, and the results are compared with the results of [Oztop et al. \(2011\)](#) for the same problem. It is observed that very good agreements exist between the two results.

The solution procedure is repeated until the following convergence criterion is satisfied

$$\text{error} = \frac{\sum_{j=1}^{j=M} \sum_{i=1}^{i=N} |\lambda^{n+1} - \lambda^n|}{\sum_{j=1}^{j=M} \sum_{i=1}^{i=N} |\lambda^{n+1}|} < 10^{-7}. \quad (18)$$

Here, M and N denotes the number of grid points in x and y directions, respectively. N is the number of iteration and  $\lambda$  denotes any scalar transport quantity.

To verify grid independence, nine different grid sizes are tested from 21 × 21, 31 × 31, 41 × 41, 51 × 51, 61 × 61, 71 × 71, 81 × 81, 91 × 91 and 101 × 101. Average Nusselt number of the hot wall is obtained for each grid size as shown in fig 2.

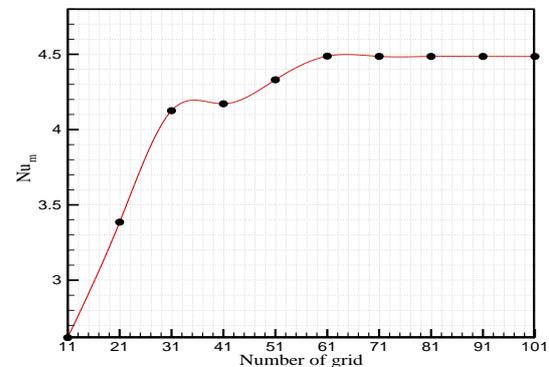


Figure 2: Mesh grid validation

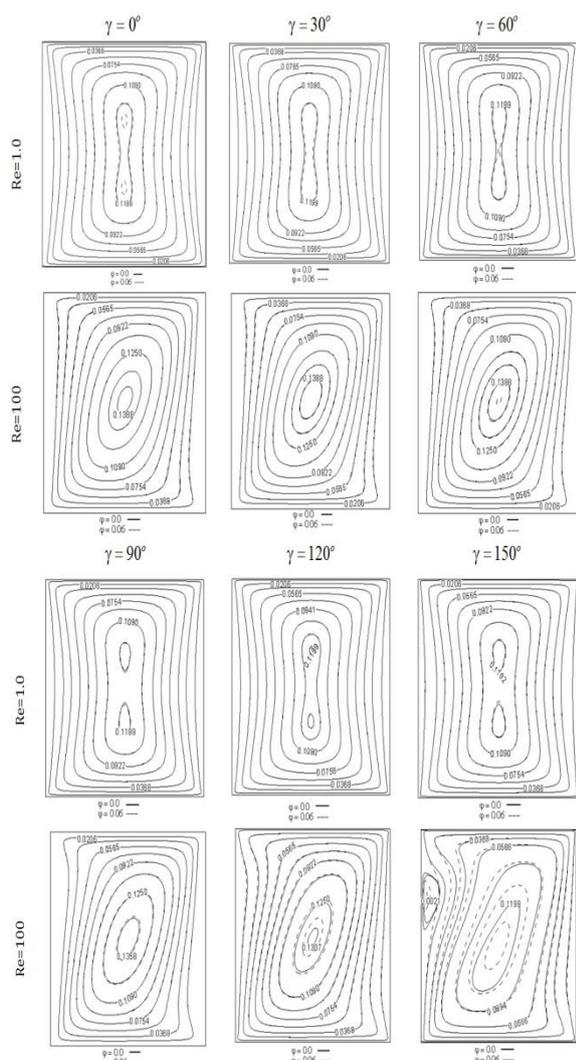
## RESULTS AND DISCUSSION

In this article, mixed convection in a square cavity with top lid has been investigated. The left and right sides of cavity kept at constant temperature while horizontal sides are insulated.

Fluid flow of  $Al_2O_3$  nanoparticles for Re between 1 to 300, Ri between 0.1 to 1, and different  $\varphi$  and  $\gamma$  are calculated and discussed. Plots of the streamlines for Ri=1, and different Re and different  $\gamma$  for nanofluid ( $\varphi=0.06$ ) and base fluid are shown in Figures (2).

As can be seen in these figures, by increasing Re in a constant  $\gamma$ , intensity of the streamlines

becomes more. Therefore, mass flow rate between two specific points increases, thus, This phenomenon leads to increasing the rate of heat transfer. Increasing  $\varphi$  does not have significant effect on the streamlines. However in some figures related to  $Re=100$ , the streamlines are becomes close to the wall. When  $\gamma=150^\circ$ , the fluid is heated and consequently flows upward while the movement of the walls causes the nanofluid to move downward.



**Figure 3:** Streamlines for various Re and for  $Ri=1$

Therefore in the lower area of the heated wall at this slope, a small vortex is formed. By increasing  $\varphi$ , the streamlines tend to get closer to the walls. So the amount of mass flow rate that is flowing near to the heat source increases thus increasing the rate of heat transfer. For  $Re=100$  and  $\gamma=150^\circ$ , by increasing  $\varphi$ , the size of the formed vortex near the heat source surface decreases.

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