

DETERMINATION OF DISCHARGE COEFFICIENT IN INCLINED RECTANGULAR SHARP CRESTED WEIRS USING EXPERIMENTAL AND NUMERICAL SIMULATION

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ABSTRACT: The sharp-crested weir is the most commonly used device in channels for flow measurement and regulation, due to its simplicity. Attempts have been made to study flow over different shapes of normal, side and oblique weirs. This research carried out for development of inclination-flow model to measure flow over inclined rectangular weir. In the present study, four different inclined weirs are considered for validation purposes. The calculated over flow parameters are compared against 2D numerical data as well as experimental studies which is showed that: The higher inclined-weir discharging index associated with angle will help to improve discharge coefficient.

KEYWORDS: Sharp-crested weir, rectangular weir, inclined weir, 2D numerical.

INTRODUCTION

A weir is defined as an obstruction in open channel that water must flow over and is used as an indirect method for obtaining the flow rate based on the weir geometry and head on the weir crest ([King and Brater, 1963](#)). In order to reach the maximum efficiency in the discharge, a sharp crested weir is mostly used. It allows the water to fall cleanly away from the weir like a free jet without side contraction. The length of the crest in the direction of the flow consists of a thin overflow plate. Discharge measurements can be obtained with high accuracy and the sharp-crested weirs are often employed in hydraulic laboratories and industry. The use of sharp-crested weirs in this research is to determine the flow characteristics as well as to estimate discharge coefficient. The general weir equation for weirs with horizontal crests is given in the following equation ([King and Brater, 1963](#)):

$$Q = C_d b h_w^{1.5} \quad (1)$$

In which, Q is volumetric flow rate, h_w is the head on the weir crest, C_d is a weir discharge coefficient and b is the length of the weir crest. The discharge coefficient depends on the effect of viscosity, the velocity distribution in the approach section, and capillarity effect of water, but it is most easily found by empirical methods ([Rouse, 1950](#)). This equation is useful to determine volumetric flow rate, Q , by simply measuring the head on the weir at an upstream location, assuming one has previously found the correct value of C_d for the weir being used.

Numerical and experimental studies of flow over weirs have a number of applications especially in the analyses of flow over common types of civil engineering structures. The problem of flow over weirs has been extensively studied experimentally. Most of the experimental works were performed to understand the flow characteristics of these weirs as well as the determination of the coefficients of discharge under free submerged flow conditions ([Fritz and Hager, 1998](#)).

A review of the literature shows that little effort has so far been made to study the numerical modeling of flow over sharp crested weirs, particularly for the establishment of relationship between the discharge coefficient and ratio of the head over a weir to the weir height.

[Muraldihar, \(1965\)](#) conducted experiments on weirs of finite crest width. [Swamee et al., \(1994\)](#) developed equation for elementary discharge coefficient for rectangular side weir. [Jain and Fischer, \(1982\)](#) have studied weirs with crest oblique to the flow axis in the channel. Multi fold skew weirs called labyrinth weirs were developed and tested by [Hay and Taylor, \(1970\)](#). They presented results in the form of curves between discharge coefficient and weir head to weir crest ratio and compared the results with normal weir. Tullis studied Labyrinth weir having trapezoidal plan forms and presented results in the form of curves between discharge coefficient and E/w with angle as the third parameter. Rammamurthy has shown that with the increase in downstream slope for circular crested weir, discharge coefficient can be improved. Talib Mansoor has developed

equation for elementary discharge coefficient for rectangular skew weirs.

Shesha Prakeash and Shivapur have studied the variation of discharge coefficient with the angle of inclination of rectangular weir plane with respect to normal position of plane of weir for sharp-crested triangular weir. The majority of these researchers have used other type of crested weirs to calibrate the weir which is under investigation, limiting the accuracy of the results obtained.

[Chatila and Tabara, \(2004\)](#) simulated flow on an ogee spillway by finite element method. They used $k-\epsilon$ two-equation model to simulate turbulent flow. The used network has been dynamic meshes. Water surface profile was compared with physical model and showed a suitable accommodation.

[Sohrabi *et al.*, \(2011\)](#) studied flow on trapezoidal weir using physical linear fit and then simulated it by fluent numerical model. In simulating flow with turbulent flow models, Realizable model of $K-\epsilon$ model had the lowest error. The mean relative error in simulating upstream water depth and water depth on crest have been 3% and 3.5% respectively.

[Arvanaghi and Nasehi Oskuei, \(2013\)](#) computed discharge coefficient for sharp-crested weirs using Fluent CFD model. The effect of height of weir, Froude number and Reynolds number on discharge were studied by experimental and numerical data. The result showed that for a special value of these parameters, discharge coefficient bows to constant scalar of 0.7.

[Afshar and Hoseini, \(2013\)](#) evaluated flow on the circular broad-crested weir in the laboratory. According to experimental results they simulated flow using CFD numerical method 3 dimensionally. The results of their calculations corresponded well with experimental data. Also the results showed that RNG $K-\epsilon$ model has lowest error.

[Maghrebi *et al.*, \(2012\)](#) simulated flow on a rectangular broad-crested weir numerically using Flow-3D software. Numerical simulation showed that Flow-3D software can efficiently simulate flow pattern. The results also showed an appropriate prediction of water surface profile and hydraulic jump in that hydraulic structure.

[Hoseini *et al.*, \(2013\)](#) estimated discharge coefficient for circular broad-crested weir in trapezoidal channels using CFD model. Afterwards the obtained values of numerical model were compared for Froude number, Reynolds number and channel slope with experimental data. Mean error value between numerical and experimental data was 5.13% and the results of two models had a good match.

[Mahmodinia *et al.*, \(2014\)](#) simulated flow capacity and the free water surface pattern in submerged side spillway with different length using numerical modeling and compared the results with laboratory measures. Comparison between numerical and physical results, showed an appropriate accuracy in prediction.

[Zahiri *et al.*, \(2014\)](#) conducted mathematical modeling of flow discharge on combined sharp-crested weirs. The suggested model improved discharge calculating since the absolute average error was about 3.8 percent, while previous models had an error more than 12.5 percent.

[Namaee *et al.*, \(2014\)](#) simulated the 3 dimensional flows on a broad-crested side spillway in a numerical way. RNG $K-\epsilon$ model was used to simulate turbulent and fluid volume model to find free water surface. The simulation results were evaluated using experimental data and showed that existing numerical methods using RANS are useful in designing side weirs.

MATERIALS AND METHODS

2.1. Physical Model

In the present research, sharp-crested rectangular weir of 250mm crest length is fixed inclined with respect to the bed of channel. The conventional method of volumetric measurement is used to final actual discharge over the weir.

Experiments were carried on inclined rectangular weir fixed normal to the flow direction 0° , 15° , 30° and 45° inclinations with respect to the normal plane along the flow axis. As can be seen in Figure 1, the flow length along the plane of the weir increases with increase in inclination with the normal plane which increases the discharging capacity of the weir.

The experiments were conducted at the hydraulic laboratory at Tabriz University, Iran. The experiments were conducted in a horizontal rectangular channel 25 cm wide and 70 cm high with vertical plexiglass sides. The total length of channel is 12 m, with the front side of glass and the bottom and back side of black PVC.

The inclined rectangular weir is made of 5mm plexiglass with height of 20 cm and was located at 5 m downstream of the inlet. Thus, an excellent, smooth and wave less flow was obtained that could be analyzed accurately. The experimental set up is shown in Figure 2.

Water is supplied to the channel by an inlet valve provided on supply pipe.

The head over the weir is measured using an electronic point gage located at a distance of about 2 meters on upstream of inclined rectangular weir.

Output stream of channel first enters to a tank of stream controller, then passes through calmer

tank and finally, the flow passes over a triangular weir with calibrated vertex angle of 53 degrees. This weir is used to measuring volume of flow that is passed over in a weir.

An ultrasonic point gage level meter is used to measure depth of flow behind of triangular weir.

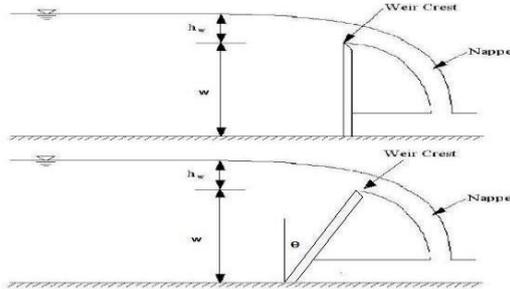


Figure 1: Section of weir along the flow

Water depth in upstream and downstream of weir was measured with another ultrasonic level meter. After running through the experimental setup, water is collected in an underground sump from which it is recirculated by pump by lifting it back to overhead tank. The present investigation was carried out on the range of variables shown in Table-1.



Figure 2: Experimental setup

Table 1: Range of variables studied

| Position of the weir | Angle of inclination (Degree) | | | |
|-------------------------|-------------------------------|---------------|----------------|----------------|
| | 0 | 15 | 30 | 45 |
| Actual discharge (CMS) | 0.009 to 0.225 | 0.01 to 0.232 | 0.012 to 0.238 | 0.013 to 0.246 |
| Head over the crest (m) | 0.02 to 0.22 | | | |
| Number of runs | 11 | 11 | 11 | 11 |

2.2. Numerical Model

In this study, numerical modeling is carried out by *Fluent v. 6.2*. *Fluent* is one of the powerful and common CFD commercial software. It first transforms the governing equations to the algebraic equations by finite volume method then solves them.

Fluent has the ability to solve 2D and 3D problems of open channel flow, confined conduit flow and sediment transport by advanced turbulence models. Also it is possible to solve continuum and momentum equations so called Navier-Stocks equations around sharp-crested weirs. The governing equations of our interest are unsteady incompressible 2-dimensional continuity and Reynolds-averaged Navier-Stocks equations (RANS) for liquid and air ([Liu et al. 2002](#)).

$$\frac{\partial \rho}{\partial t} + \frac{\partial (\rho u_i)}{\partial x_i} = 0 \quad (2)$$

$$\frac{\partial (\rho u_i)}{\partial t} + \frac{\partial (\rho u_i u_j)}{\partial x_j} = -\frac{\partial p}{\partial x_i} + \frac{\partial}{\partial x_j} \left[\mu \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} - \frac{2}{3} \delta_{ij} \frac{\partial u_k}{\partial x_k} \right) \right] + \frac{\partial (\rho \overline{u_i u_j})}{\partial x_j} \quad (3)$$

Where, ρ = fluid density, u = velocity components, x = space dimensions, t = time, p = hydrostatic pressure, μ = dynamic viscosity, $\overline{u_i u_j}$ = Reynolds stress tensor and δ_{ij} = crooner delta.

There are different methods to solve RANS. The Reynold's stress model (RSM) is a higher level elaborate turbulence model. In RSM, the eddy viscosity approach has been discarded and the Reynold's stress is directly computed. The RSM involves calculation of individual Reynolds stress, $\overline{\rho U'_i U'_j}$ using differential transport equations that is written as follows:

$$\begin{aligned} \frac{\partial}{\partial t} (\overline{\rho U'_i U'_j}) + \frac{\partial}{\partial x_k} (\overline{\rho \rho U'_i U'_j U'_k}) = & -\frac{\partial}{\partial x_k} \left[\overline{\rho U'_i U'_j U'_k} + p' (\delta_{kj} U'_i + \delta_{ik} U'_j) \right] + \frac{\partial}{\partial x_k} \left(\mu \frac{\partial}{\partial x_k} (\overline{U'_i U'_j}) \right) \\ & - \overline{\rho (U'_i U'_k \frac{\partial U'_j}{\partial x_k} + U'_j U'_k \frac{\partial U'_i}{\partial x_k})} - \rho \beta (g_i U'_j \theta + g_j U'_i \theta) + p' \left(\frac{\partial U'_i}{\partial x_i} + \frac{\partial U'_j}{\partial x_j} \right) - 2\mu \frac{\partial U'_i}{\partial x_k} \frac{\partial U'_j}{\partial x_k} \\ & - 2\rho p_k (\overline{U'_j U'_m \epsilon_{ikm}} + \overline{U'_i U'_m \epsilon_{jkm}}) + S_{user} \end{aligned} \quad (4)$$

Where, ϵ = energy dissipation rate and the other parameters are model coefficients.

To start *Fluent*, we first need to specify channel geometry and then generate a mesh for it. *Gambit* software is used to do this. Because four

different inclinations are defined for experimental models of weirs, we need to design an especial geometry and mesh for each weir.

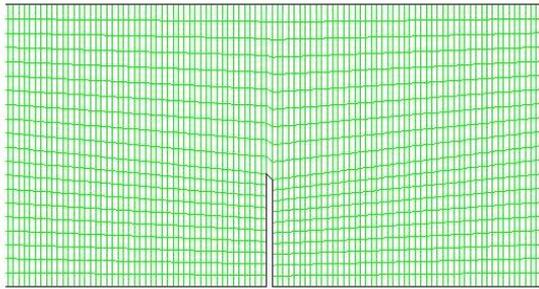


Figure 3: Simple of meshed vertical weir model

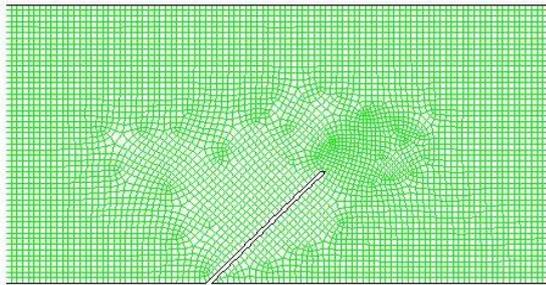


Figure 4: Simple of meshed inclined weir model

RESULTS AND DISCUSSION

During each test, the water depth upstream and downstream of the weir were measured after that the flow was balanced. Then the water surface profile in upstream and drawn stream was compared with *fluent* results. Figures 5 and 6 depict the water surface profile for vertical and 45 degree inclined weirs respectively, which have run in the *Fluent*. **Error! Reference source not found.** Figure 7 also shows a comparison between numerical and experimental profiles. The results show the good conformity of numerical result by experimental ones.

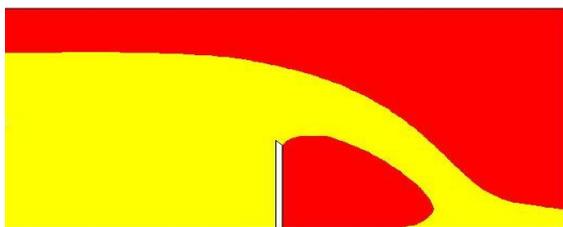


Figure 5: Water surface profile modeled by Fluent (Vertical Weir)

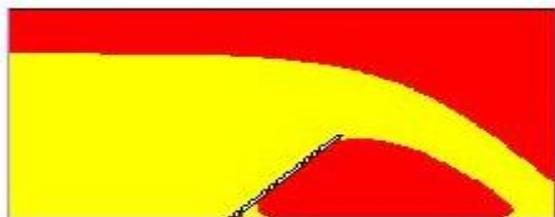


Figure 6: Water surface profile modeled by Fluent (45 degree inclined weir)

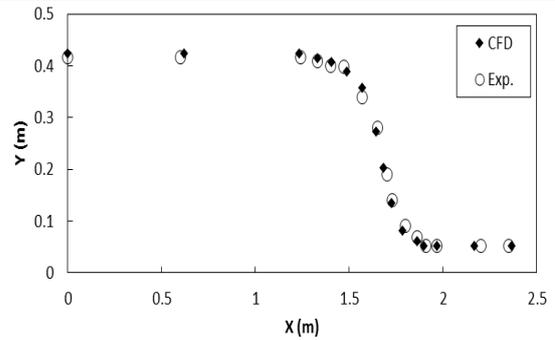


Figure 7: Comparison of experimental and numerical water surface profile (45 degree inclined weir)

A plot of non-dimensional discharge versus non-dimensional head for various inclinations of plane of rectangular weir has been shown in Figure 8. It shows that the discharge increases with increase in inclination angle. The discharge for flow through rectangular weir is given by:

$$Q = \frac{2}{3} \sqrt{2gb} h_w^{1.5} \tag{5}$$

Where, all parameters have been obtained in metric system. The discharge-head equation can be expressed as:

$$Q = KH^n \tag{6}$$

A head-discharge plot is shows that discharge is increasing by inclining the weirs in same heads.

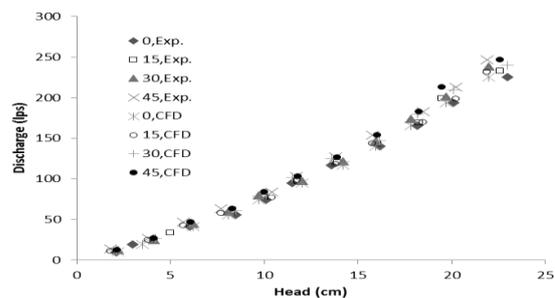


Figure 8: Head-discharge plot for various values of inclinations

Using regression analysis, the actual head-discharge data values were fitted with the exponential equation (table 2).

Table 1: Calibration head-discharge equation for various angles

| No | Inclination angle | Equation |
|----|-------------------|--------------------|
| 1 | 0 | $Q=3.826H^{1.302}$ |
| 2 | 15 | $Q=4.375H^{1.273}$ |
| 3 | 30 | $Q=4.457H^{1.262}$ |
| 4 | 45 | $Q=5.331H^{1.216}$ |

According to above equations, it is understood that with increasing in the inclination angle, K (discharge coefficient) is increased too. Figures 9 to 12 represent dimensionless height of weir (H/w) effect on discharge coefficient (C_d).

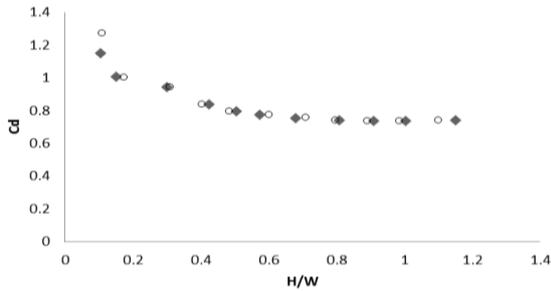


Figure 9: Comparison of numerical and experimental C_d for different H/W (0 degree inclination)

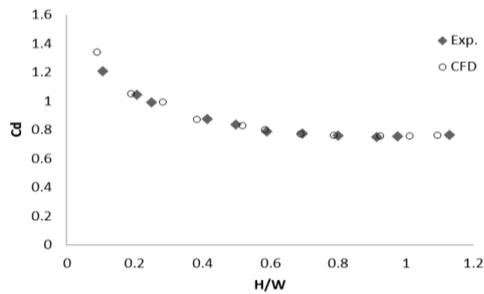


Figure 10: Comparison of numerical and experimental C_d for different H/W (15 degree inclination)

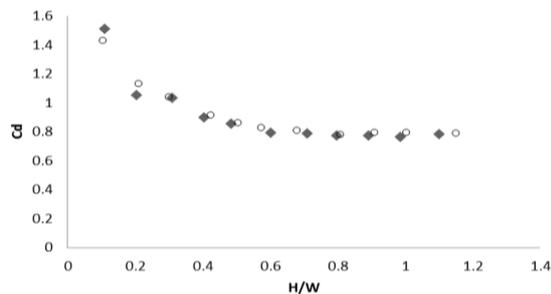


Figure 11: Comparison of numerical and experimental C_d for different H/W (30 degree inclination)

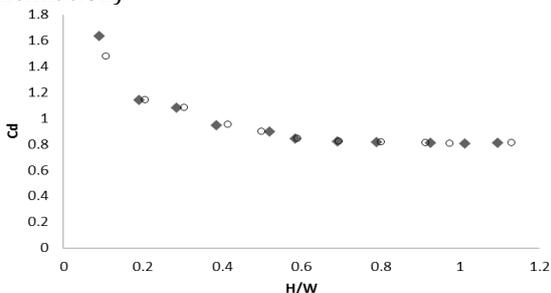


Figure 12: Comparison of numerical and experimental C_d for different H/W (45 degree inclination)

A coefficient of discharge of 15, 30 and 45 degree inclined weirs to corresponding vertical weir ratio versus dimensionless height of weir is shown in Figure 13.

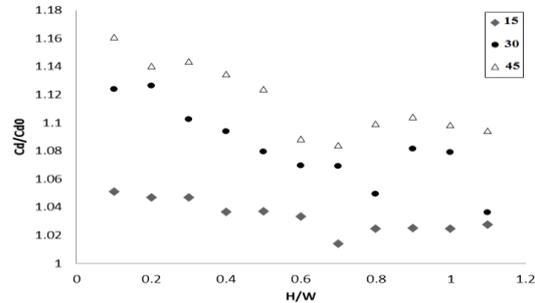


Figure 133: Comparison of C_d/C_{d0} for different H/W (15, 30 and 45 degree inclination)

CONCLUSION

Measurement of discharge in open channel is one of the main concerns in hydraulic engineering. Sharp crested weirs are used to obtain discharge in open channels by solely measuring the water head upstream of the weir. The goal of this research was to evaluate and validate the flow over inclined sharp-crested weirs.

Following conclusions were drawn based on the experimental and CFD investigations by the authors.

- The discharging coefficient of the weir increases with the increase in inclination of the plane of weir.
- Larger amount of flow is possible in the inclined sharp-crested weirs relative to the normal vertical sharp-crested weir.

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