JOURNAL OF CURRENT RESEARCH IN SCIENCE (ISSN 2322-5009)
CODEN (USA): JCRSDJ
2014, Vol. 2, No. 3, pp: 422-427
Available at www.jcrs010.com

ORIGINAL ARTICLE

SOLUTIONS OF NANO BOUNDARY LAYERS OVER STRETCHING SURFACES BY USING THE LAPLACE ADOMIAN DECOMPOSITION METHOD

Sima Samadpoor[1], Hadi Roohani Ghehsareh[1], Saeid Abbasbandy[2]
1- Department of Mathematics, Buin Zahra Branch, Islamic Azad University (I.A.U), Buin Zahra, Iran
2- Department of Mathematics, Imam Khomeini International University, Qazvin, Iran

ABSTRACT: In this paper, an interesting physical model which describes the nano boundary layer flows with Navier boundary condition, is investigated. By using some suitable similarity transformations, the governing partial differential equations are transformed to a nonlinear ordinary differential problem with boundary conditions. An efficient semi-analytical method, the Laplace Adomian Decomposition Method (LADM) coupled with Padé approximation, is formulated and used to approximate the similarity solutions of the problem. Two cases of the model, when the viscous flows over a two-dimensional stretching surface and an axisymmetric stretching surface are investigated. The presented results through the tables and figures show the efficiency and accuracy of the proposed semi-analytical method. Finally, some results are given to show the effects of the model parameters (suction parameter and slip parameter) on the fluid velocity and on the tangential stress.

KEYWORD: Boundary-layer problems, Nano boundary layers, Laplace Adomian Decomposition Method, Padé approximation.

INTRODUCTION

The study of boundary layer flow due to viscous fluids has gained considerable interest in the last decades (Schlichting, 1979; White, 2006). By using the boundary layer concept, many mathematical results agree well with the experimental observations. It is well known that, in the classical boundary layer theory the condition of no-slip near solid walls is usually applied. That is, the fluid velocity component is assumed to be zero relative to the solid boundary (Van Gorder *et al.,* 2010). But this theory is not true for fluid flows at the micro and nano scale. It is found that a certain degree of tangential slip must be allowed. Recently, an interesting model which describes the nano boundary layer flows with Navier boundary condition has been successfully introduced and investigated by authors (Van Gorder *et al.,* 2010; Wang, 2002; Wang, 2009).

In investigation of boundary layers problems, usually the governing system of partial differential equations would converted into a non-linear ordinary boundary value problem over a semi-infinite interval by a suitable variables transformation. Many numerical and analytical methods have been applied to approximate the similarity solution of the problems in boundary layers flow (Rashidi and Erfani, 2011; Abbasbandy and Hayat, 2009; Hayat *et al.,* 2009; Abbasbandy and Shivanian, 2010; Abbasbandy and Roohani Ghehsareh, 2012; Abbasbandy and Roohani Ghehsareh, 2013; Abbasbandy *et al.,* 2012; Abbasbandy *et al.,* 2013; Samadpoor *et al.,* 2013; Abbasbandy and Roohani Ghehsareh, 2011). In recent decade, a new form of the Adomian Decomposition Method (Adomian, 1994; Adomian and Rach, 1996), namely the Laplace Adomian Decomposition Method (Khuri, 2001; Wazwaz, 2010; Ongun, 2011) has been introduced and well-used to solve many problems. Recently, an elegant combination of the Laplace Adomian Decomposition Method and the Padé approximations (Baker, 1975) has been applied for solving some boundary layer problems which involve a boundary condition at infinity (Sivakumar and Baiju, 2011; Khan and Hussain, 2011; Khan and Gondal, 2011; Soltanalizadeh *et al.,* 2013; Roohani Ghehsareh *et al.,* 2012).

In the current manuscript, the Laplace Adomian Decomposition method coupled with the Padé approximations is employed for obtaining the similarity solution of the nano boundary layer flows with Navier boundary condition. As in Van Gorder *et al.,* (2010); Wang, (2002); Wang, (2009) the viscous flow due to a stretching surface with both surface slip and suction (or injection) is considered.

PROBLEM DESCRIPTION

In this section, we will investigate an interesting model of nonlinear problem that describes the viscous flow due to a stretching surface with both surface slip and suction (Van Gorder *et al.,* 2010; Wang, 2009). As given in Van Gorder *et al.*, (2010) and Wang, (2009), let (u, v, w) be the velocity components in the (x, y, z) directions, respectively, and let p be the pressure. Then the Navier-Stokes equations for the steady viscous fluid flow can be shown as:

$$uu_x + vu_y + wu_z = -\frac{p_z}{\rho} + \nu(u_{xx} + u_{yy} + u_{zz}),$$
$$uv_x + vv_y + wv_z = -\frac{p_y}{\rho} + \nu(v_{xx} + v_{yy} + v_{zz}),$$
$$uw_x + vw_y + ww_z = -\frac{p_x}{\rho} + \nu(w_{xx} + w_{yy} + w_{zz}),$$

where ν and ρ are the kinematic viscosity and the density of the fluid, respectively. The continuity equation can be shown as:

$$u_x + u_y + u_z = 0.$$

As in Van Gorder *et al.*, (2010) and Wang, (2009), we take the velocity on the stretching surface (on the plane $z = 0$) as:

$$u = ax, \quad v = (m-1)ay, \quad w = 0,$$

Where, $a > 0$ is the stretching rate of the sheet and m is a parameter describing the type of stretching. When $m = 1$, we have two-dimensional stretching, while for $m = 2$ we have axisymmetric stretching. To simplify the governing equations we use the similarity variable $\tau = \sqrt{a/\nu z}$ and the similarity functions:

$$u = axf'(\tau), \quad v = (m-1)ayf'(\tau), \quad w = -m\sqrt{a\nu}f(\tau).$$

The continuity equation is satisfied automatically and the Navier-Stokes equations become:

$$\frac{d^3f}{d\tau^3} - (\frac{df}{d\tau})^2 + mf\frac{d^2f}{d\tau^2} = 0,$$

As there is no lateral pressure gradient at infinity. On the surface of the stretching sheet, the velocity slip is assumed to be proportional to the local shear stress:

$$u - ax = N\rho\nu\frac{\partial u}{\partial z} < 0, \qquad v - (m-1)ay = N\rho\nu\frac{\partial v}{\partial z} < 0, \qquad \textbf{(1)}$$

Where, N is a slip constant. By using the similarity transform, (1) can be written as:

$$f'(0) - 1 = Kf''(0),$$

Where, $K = N\rho\sqrt{a\nu} > 0$ is a non-dimensional slip parameter. From (1), we have that $f''(0) < 0$. Given a suction velocity of $-W$ on the stretching surface, we have the boundary condition:

$$f(0) = \lambda,$$

Where, $\lambda = W/(m\sqrt{a\nu})$. Since there is no lateral velocity at infinity, we have the condition:

$$\lim_{\tau\to\infty} f'(\tau) = 0.$$

With these considerations in mind, we set out to obtain solutions to the nonlinear ordinary differential equation:

$$\frac{d^3f}{d\tau^3} - (\frac{df}{d\tau})^2 + mf\frac{d^2f}{d\tau^2} = 0, \qquad \textbf{(2)}$$

With boundary conditions:

$$f(0) = \lambda, \quad f'(0) = 1 + Kf''(0), \quad \lim_{\tau\to\infty} f'(\tau) = 0. \qquad \textbf{(3)}$$

Now our interesting is to obtain the similarity solution of (2) with boundary conditions (3). Wang, (2009) has investigated the problem (2), and has shown the existence and uniqueness results for both the two-dimensional stretching surface and the axisymmetric stretching surface of the problem and also has presented a closed form of the similarity solution for (2), when $m = 1$ in the following form:

$$f(\tau) = \lambda + (C - \lambda)(1 - e^{-c\tau}), \qquad \textbf{(4)}$$

Where, C is the maximal root of the equation:

$$KC^3 + (1 - \lambda K)C^2 - \lambda C - 1 = 0.$$

Recently, Van Gorder *et al.*, (2010) have been solved this problem analytically by using the homotopy analysis method. Rashidi and Erfani, (2011) have applied the differential transform method (DTM)-Padé technique for solving this problem. In this work we consider two geometric situations, a two-dimensional stretching surface $(m = 1)$ and an axisymmetric stretching surface$(m = 2)$.

FORMULATION OF LAPLACE ADOMIAN DECOMPOSITION METHOD

In this section we will apply the Laplace Adomian Decomposition Method for solving the nonlinear ordinary differential equation (2) with boundary conditions (3). For these purpose, in first step by taking Laplace transformation $(\mathcal{L})$ to both sides of Eq.(2) we have:

$$\mathcal{L}\{f(\tau)\} = \frac{1}{s^3}f''(0) + \frac{1}{s^2}f'(0) + \frac{1}{s}f(0) + \frac{1}{s^3}\mathcal{L}\{f'(\tau)^2\} - \frac{m}{s^3}\mathcal{L}\{f(\tau)f''(\tau)\}, \qquad \textbf{(5)}$$

With

$$f(0) = \lambda, \quad f'(0) = 1 + K\alpha, \quad f''(0) = \alpha, \qquad \textbf{(6)}$$

Notice that, α is a constant to be determined. By applying the conditions (6) into (5) we get:

$$\mathcal{L}\{f(\tau)\} = \frac{1}{s^3}\alpha + \frac{1}{s^2}(1 + K\alpha) + \frac{1}{s}\lambda + \frac{1}{s^3}\mathcal{L}\{f'(\tau)^2\} - \frac{m}{s^3}\mathcal{L}\{f(\tau)f''(\tau)\}. \qquad \textbf{(7)}$$

Now based on the Laplace Adomian Decomposition Method, we represent the solution of (7) as an infinite series given below:

$$f(\tau)=\sum_{n=0}^{\infty} f_n(\tau), \qquad \textbf{(8)}$$

Where the components $f_n(\tau)$ will be determined recurrently and also the nonlinear terms $f'(\tau)^2$ and $f(\tau)f''(\tau)$ can be usually decomposed by infinite series of the so-called Adomian polynomials (Adomian, 1994; Adomian and Rach, 1996):

$$N_1(f)=f'(\tau)^2=(\sum_{n=0}^{\infty} f_n(\tau))'^2=\sum_{n=0}^{\infty} A_n,$$
$$N_2(f)=f(\tau)f''(\tau)=(\sum_{n=0}^{\infty} f_n(\tau))(\sum_{n=0}^{\infty} f_n(\tau))''=\sum_{n=0}^{\infty} B_n,$$

Where the Adomian polynomials A_n and B_n are computed from:

$$A_n=\frac{1}{n!}[\frac{d^n}{d\xi^n}N_1(\sum_{i=0}^{\infty}\xi^i f_i(\tau))]_{\xi=0},$$
$$B_n=\frac{1}{n!}[\frac{d^n}{d\xi^n}N_2(\sum_{i=0}^{\infty}\xi^i f_i(\tau))]_{\xi=0},$$

First few Adomian polynomials of these nonlinear terms are, $A_0={f_0}'^2, A_1=2{f_0}'{f_1}', A_2=f_{1'}^2+2f_{0'}f_{2'}$ and $B_0=f_0f_{0''}, B_1=f_1f_{0''}+f_0f_{1''}, B_2=f_0f_{2''}+f_2f_{0''}+f_1f_{1''}$. By substituting the assumed solution and Adomian polynomials into (7) we get:

$$\mathcal{L}\{\textstyle\sum_{n=0}^{\infty} f_n(\tau)\}=\frac{1}{s^3}\alpha+\frac{1}{s^2}(1+K\alpha)+\frac{1}{s}\lambda+\frac{1}{s^3}\mathcal{L}\{\sum_{n=0}^{\infty} A_n\}-\frac{m}{s^3}\mathcal{L}\{\sum_{n=0}^{\infty} B_n\} \qquad \textbf{(9)}$$

On the other hand, if we let $K(s)=\frac{1}{s^3}\alpha+\frac{1}{s^2}(1+K\alpha)+\frac{1}{s}\lambda$ represents the term arising from prescribe initial conditions, then based on the modified Laplace decomposition method (Khan and Hussain, 2010), the function $K(s)$ can be decompose into three parts, i. e. $K(s)=K_0(s)+K_1(s)+K_2(s)$. Now for computing $f_n, (n\geq 0)$, firstly we compare both sides of the equation (9) and then apply the inverse Laplace transform $\mathcal{L}^{-1}$. Then we get the following iterative algorithm:

$$f_0=\mathcal{L}^{-1}\{\frac{1}{s^3}\alpha\},$$
$$f_1=\mathcal{L}^{-1}\{\frac{1}{s^2}(1+K\alpha)+\frac{1}{s^3}\mathcal{L}\{A_0\}-\frac{m}{s^3}\mathcal{L}\{B_0\}\},$$
$$f_2=\mathcal{L}^{-1}\{\frac{\lambda}{s}+\frac{1}{s^3}\mathcal{L}\{A_1\}-\frac{m}{s^3}\mathcal{L}\{B_1\}\},$$
$$f_i=\mathcal{L}^{-1}\{\frac{1}{s^3}\mathcal{L}\{A_{i-1}\}-\frac{m}{s^3}\mathcal{L}\{B_{i-1}\}\}, i=3,4,5,\ldots \qquad \textbf{(10)}$$

Now in the equation, firstly we get the initial term f_0, by using inverse Laplace transform. Then by the known value of f_0, we obtain the f_1. By continuing this process, we can find the successive terms. Then:

$$f_0=1/2\,\alpha\tau^2,$$
$$f_1=(1+K\alpha)\tau-\frac{1}{120}\alpha^2(-2+m)\tau^5,$$
$$f_2=\lambda\tau-\frac{1}{40320}(-11\,\alpha^2\tau^4 m+10\,\alpha^2\tau^4+1680+1680\,K\alpha)(-2+m)\tau^4\alpha,$$
$$f_3=\frac{1}{13305600}\tau^3(2217600+4435200\,K\alpha+52800\,\alpha^2\tau^4-84480\,\alpha^2\tau^4 m$$
$$+2217600\,K^2\alpha^2+52800\,K\alpha^3\tau^4-84480\,K\alpha^3\tau^4 m+200\,\alpha^4\tau^8$$
$$-532\,\alpha^4\tau^8 m+466\,\alpha^4\tau^8 m^2+1108800\,\alpha\tau\lambda-554400\,\tau m\alpha\lambda$$
$$-125\,\tau^8 m^3\alpha^4+29040\,\tau^4\alpha^2 m^2+29040\,\tau^4 m^2\alpha^3 K),$$
$$f_4=\frac{1}{29059430400}\tau^3(9686476800\,K\alpha\lambda+9686476800\,\lambda-3003000\,\tau^7\alpha^3 m^3$$
$$+403603200\,\alpha\tau^3+13200\,\alpha^5\tau^{11}+4804800\,\alpha^3\tau^7+403603200\,K^2\alpha^3\tau^3$$
$$-49744\,\alpha^5\tau^{11}m+9299\,\tau^{11}\alpha^5 m^4+121080960\,\tau^3\alpha m^2+11195184\,\alpha^3\tau^7 m^2$$
$$+115315200\,\alpha^2\tau^4\lambda+4804800\,K\alpha^4\tau^7+69480\,\alpha^5\tau^{11}m^2-42552\,\alpha^5\tau^{11}m^3$$
$$+807206400\,K\alpha^2\tau^3-484323840\,\alpha m\tau^3-12780768\,\alpha^3\tau^7 m$$
$$-484323840\,K^2\alpha^3 m\tau^3$$
$$-968647680\,mK\alpha^2\tau^3-12780768\,K\alpha^4\tau^7 m+11195184\,K\alpha^4\tau^7 m^2$$
$$+63423360\,\tau^4\alpha^2 m^2\lambda$$
$$-184504320\,\alpha^2\tau^4 m\lambda+121080960\,\tau^3\alpha^3 m^2K^2+242161920\,\tau^3\alpha^2 m^2 K-$$
$$3003000\,\tau^7\alpha^4 m^3 K), \qquad \textbf{(11)}$$

By obtaining the components $f_i(\tau)$, for $i=0,1,2,3,\ldots.,$ the approximate explicit candidate solution of equation can be found from equation (8). The approximate analytic solution for the second iteration process is:

$$f(\tau)=\frac{1}{2016}\tau^8\alpha^3-\frac{2}{315}\tau^7K\alpha^3 m-\frac{1}{24}\tau^4\alpha^2Km$$
$$+\frac{1}{12}\tau^4\alpha+\frac{11}{5040}\tau^7m^2\alpha^3K-\frac{1}{24}\tau^4m\alpha\lambda+\tau$$
$$+\frac{11}{40320}\tau^8\alpha^3m^2-\frac{1}{1260}\tau^8\alpha^3m-\frac{1}{120}\alpha^2\tau^5m$$
$$+\frac{1}{60}\alpha^2\tau^5+\tau K\alpha$$
$$-\frac{2}{315}\tau^7\alpha^2m+1/3\,\tau^3K\alpha+1/12\,\tau^4\alpha^2K-1/24\,\tau^4\alpha m+\frac{11}{5040}\tau^7\alpha^2m^2$$
$$-\frac{5}{532224}\tau^{11}m^3\alpha^4+1/12\,\tau^4\alpha\lambda+\frac{233}{6652800}\tau^{11}\alpha^4m^2-\frac{19}{475200}\tau^{11}\alpha^4m$$
$$+\frac{1}{252}\tau^7K\alpha^3+1/6\,\tau^3K^2\alpha^2+1/6\,\tau^3+1/2\,\alpha\tau^2+\lambda\tau+\frac{1}{252}\tau^7\alpha^2$$
$$+\frac{1}{66528}\tau^{11}\alpha^4+. \qquad (12)$$

From Equition, it is easily observed that the analytic solutions obtained through LADM are power series in the independent variable. So these solutions don't have the correct behavior at infinity according to boundary condition $f'(\infty)=0$ and these solutions cannot be directly applied. Hence to overcome this problem, it is essential to combine the series solutions, obtained by the LADM, with the Padé approximants to have the correct limit at infinity.

THE LADM-PADÉ APPROXIMATION

In this section we will briefly explain main idea of the Laplace Adomian Decomposition Method coupled with Padé approximants. The LADM-Padé approximant for problem (2) is based on the transformation of the power series obtained by the Laplace Adomian Decomposition Method into a rational function as follow:

$$[S/N](\tau)=\frac{\sum_{j=0}^{S} a_j\tau^j}{\sum_{j=0}^{N} b_j\tau^j}, \qquad \textbf{(1)}$$

Where, $b_0=1$. In order to have the correct limit at infinity according to the boundary conditions (3), one would expect that $N\geq S$. So the rational function (1) has $S+N+1$ coefficient that we may choose. If $[S/N](\tau)$ is exactly a Padé approximant, then $f(\tau)-[S/N](\tau)=O(\tau^{S+N+1})$. So under such conditions, the coefficient a_j and b_j satisfy:

$$\sum_{i=0}^{j} b_i f_{j-i} = a_j, \quad j = 0, \dots, S, \tag{2}$$

$$\sum_{i=0}^{j} b_i f_{j-i} = 0, \quad j = S+1, \dots, S+N, \tag{3}$$

Where, $b_k = 0$ if $k > N$.
From (2) and (3) we can obtain the values of $a_i (0 \leq i \leq S)$ and $b_j (1 \leq j \leq N)$.

RESULTS AND DISCUSSION

In this section, the obtained recursive process is used to compute the approximate similarity solutions of the problem (2). Here the problem with both cases, when the viscous flows over a two-dimensional stretching surface $(m = 1)$ and the viscous flows over an axisymmetric stretching surface $(m = 2)$ for some typical model parameters of λ and K will be investigated. After employing the method, it is observed that the obtained solutions have a power series form which depend on the unknown parameter $\alpha = f''(0)$. So our aim is mainly concerned with the mathematical behavior of the similarity solution $f(\tau)$ in order to determine the value of unknown parameter α. To determine this unknown value the boundary condition of the similarity solution at infinity should be used. But the approximated solutions are power series, so the boundary condition at infinity could not be used directly, hence to overcome this, the Padé approximant (1) to $f(\tau)$ would be used. Through this work we will construct only diagonal approximants $[M/M]$. Now by using the boundary condition at infinity the unknown value α can be estimated with high accuracy. Substituting this value of α in the rational approximant solution obtained for $f(\tau)$ the approximate solution is computed. In Table 1, the computed results for the shear stress at the surface $(f''(0))$ using the LADM coupled with padé approximant for the case of two-dimensional stretching $(m = 1)$ and for some values of the parameters λ when $K = 1$ are reported.

Table 1: Numerical results for the $f''(0)$ for $m = 1$, $K = 1.0$ and several values of λ

λ	$\lambda = -5$	$\lambda = -1$	$\lambda = 0$	$\lambda = 3$	$\lambda = 5$
[8,8]	-0.1423763930	-0.3176721961	-0.4326598580	-0.7565368405	-
[9,9]	-0.1423763930	-0.3176721961	-0.4301597090	-0.7548005974	-0.8261668245
[10,10]	-0.1423763930	-0.3176721961	-0.4301597090	-	-0.8344920085
[11,11]	-0.1423763930	-0.3176721961	-0.4301597090	-0.7548777087	-0.8342323713
[12,12]	-0.1423763930	-0.3176721961	-0.4301597090	-0.7548776655	-0.8342398793
[13,13]	-0.1423763930	-0.3176721961	-0.4301597090	-0.7548776658	-0.8342437219
Exact	-0.1423763930	-0.3176721962	-0.4301597090	-0.7548776662	-0.8342431843

Also in Table 2, obtained results of the $f''(0)$ for the case of two-dimensional stretching $(m = 1)$ using the presented method for some values of the parameter K when $\lambda = 2$ are given.

Table 2: Numerical results for the $f''(0)$ for $m = 1$, $\lambda = 2$ and several values of K

K	$K = 0$	$K = 1$	$K = 2$	$K = 5$
[6,6]	-2.4315842570	-0.7167128932	-0.3912480137	-0.1938046428
[7,7]		-0.6808220486	-0.4007297208	-0.1800357942
[8,8]	-2.4141981114	-0.6824502942	-0.4036691292	-0.1824700370
[9,9]	-2.4142139982	-0.6823300501	-0.4035869812	-0.1821598862
[10,10]	-2.4142139904	-0.6823277763	-0.4035564966	-0.1821735252
Exact	-2.4142135623	-0.6823278038	-0.4035565856	-0.1821716650

The reported results through these Tables show that the obtained values for the shear stress at the surface $(f''(0))$ by using the LADM-padé approximant are excellent agreement with the exact values. For the case of flow over a two-dimensional $(m = 1)$, in Figures 1 and 2 the computed results of the $f'(\tau)$ using the [10/10] LADM-Padé approximants for various values of the non-dimensional suction parameter λ when $K = 1$ and some values of the slip factor K when $\lambda = 2$ are plotted, respectively.

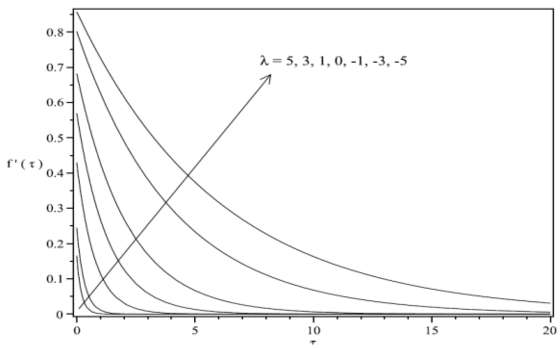

Figure 1: LADM-Padé approximate solution for $f'(\tau)$ at $K = 1.0$, $m = 1$ and for several values of λ

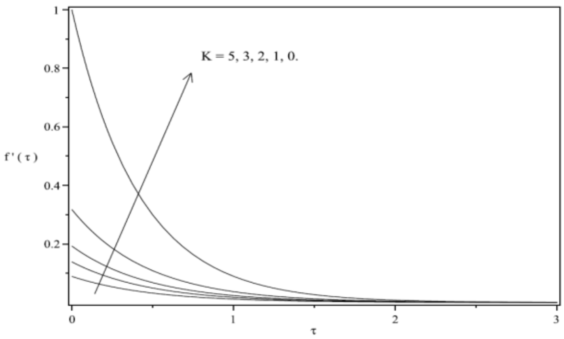

Figure 2: LADM-Padé approximate solution for $f'(\tau)$ at $m = 1$, $\lambda = 2$ and for several values of K.

From this Figure it is observed that the results are in very good agreement with boundary conditions. Here, the computed results for the viscous flows over an axisymmetric stretching surface $(m = 2)$ will be given. In Table 3, the obtained results of the skin friction coefficient (or the shear stress) $f''(0)$ by using the LADM-Padé approximants for this case $(m = 2)$ and for various values of the non-dimensional suction parameter λ and the slip factor K are reported.

Table 3: Numerical results for the shear stress at the surface $f''(0)$ for axisymmetric case $(m = 2)$, obtained by LADM-padè with [10,10]-padè approximation for various values of λ and K

S	K=0	K=1	K=3	K=5
0	-1.1737900	-0.4625271	-0.2231321	-0.1493957
2	-4.3426232	-0.8028556	-0.3078598	-0.1905162
3	-6.2394699	-0.8578612	-0.3158259	-0.1935567
5	-10.1476014	-0.9092067	-0.3225846	-0.1960791

The effect of values of the model parameters (the non-dimensional suction parameter λ and the slip factor K) on the $f'(\tau)$ computed by using the [15/15] LADM-Padé approximants are depicted in Figure 3 and 4.

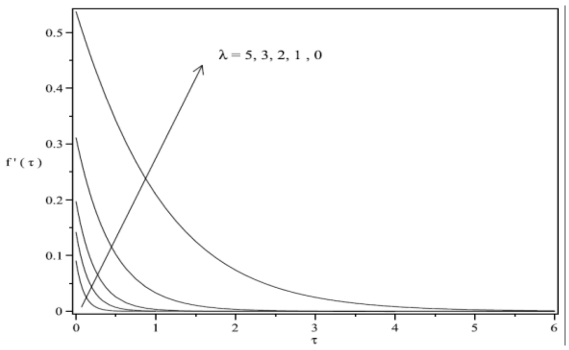

Figure 3: LADM-Padé approximate solution for $f'(\tau)$ at $m = 2$, $K = 1$ and for several values of λ.

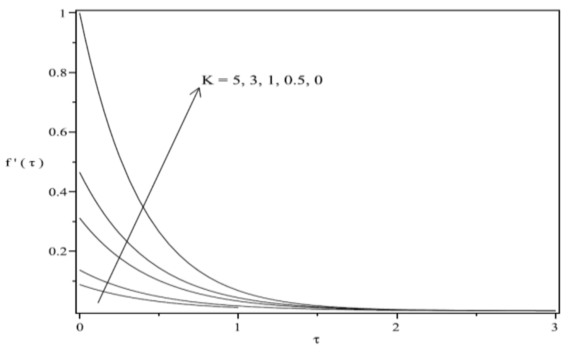

Figure 4: LADM-Padé approximate solution for $f'(\tau)$ at $m = 2$, $\lambda = 1$ and for several values of K.

Also From these Figures, clearly elucidate that the solutions obtained from the LADM-Padé approximants are in very good agreement with boundary conditions, and also show excellent agreement with the solutions already available in other literature (Van Gorder *et al.*, 2010; Wang, 2002; Wang, 2009). From Tables 1,2 and 3, it is observed that the skin friction coefficient $f''(0)$ is negative for all values of the models parameters (slip parameter K and the suction parameter λ), in both fluid flow geometries situations (flow over a two-dimensional and axisymmetric stretching surface).

CONCLUSIONS

In this work, an efficient analytical method, Laplace Adomian Decomposition Method coupled with Padé approximants is formulated and well-used to approximate the solution of the nano boundary layers over stretching surfaces. The approximate solutions are computed for various values of model parameters by using the presented method in both fluid flow geometries situations: two-dimensiona l stretching and axisymmetric stretching. The obtained results show that the simple presented method gives reasonable results with high accuracy for this problem and may be a suitable alternative for the treatment of the boundary value problems on infinite interval, specially for strong nonlinear boundary value problems.

REFERENCES

Abbasbandy S, Hayat T. Solution of the MHD Falkner-Skan flow by Hankel-Padé method. Phys Lett A 2009; 373: 731-734.

Abbasbandy S, Shivanian E. Prediction of multiplicity of solutions of nonlinear boundary value problems: Novel application of homotopy analysis method. Commun Nonlinear Sci Numer Simul 2010; 15: 3830-3846.

Abbasbandy S, Roohani Ghehsareh H. Solution of the MHD Falkner-Skan equation: Variational iteration method coupled with Pade approximation. Nonlinear Sci Lett A 2011; 2: 29-34.

Abbasbandy S, Roohani Ghehsareh H. Solutions of the magnetohydrodynamic flow over a nonlinear stretching sheet and nano boundary layers over stretching surfaces. Internat J Numer Methods Fluids 2012; 70(10): 1324-1340.

Abbasbandy S, Roohani Ghehsareh H, Hashim I. An approximate solution of the MHD flow over a non-linear stretching sheet by rational Chebyshev collocation method, Politehn. Univ Bucharest Sci Bull Ser A 2012;74(4):47-58.

Abbasbandy S, Hayat T, Roohani Ghehsareh H. A Alsaedi,MHD Falkner-Skan flow of Maxwell fluid by rational Chebyshev collocation method. Applied Mathematics and Mechanics 2013; 34(8): 921-930.

Abbasbandy S, Roohani Ghehsareh H. Solutions for MHD viscous flow due to a shrinking sheet by Hankel-Padé method. Internat. J Numer Methods Heat Fluid Flow 2013; 23: 388-400.

Adomian G. Solving frontier problems of physics: the decomposition method. Dordrecht: Kluwer Academic Publishers 1994.

Adomian G, Rach R. Modified Adomian polynomials. Math Comput Model 1996; 24(11): 39-46.

Baker GA. Essentials of Padé Approximants. Academic Press, London 1975.

Hayat T, Hussain Q, Javed T. The modified decomposition method and Padé approximants for the MHD flow over a non-linear stretching sheet. Nonlinear Analysis: Real World Applications 2009; 10: 966-973.

Khan M, Hussain M. Modified Laplace Decomposition Method. Appl Math Sci 2010; 4: 1769-1783.

Khan M, Gondal MA. A New Analytical Approach to Solve Thomas-Fermi Equation. World Applied Sciences Journal 2011; 12: 2309-2313.

Khan M, Hussain M. Application of Laplace decomposition method on semi infinite domain. Numerical Algorithms 2011; 56(2): 211-218.

Khuri SA. A Laplace decomposition algorithm applied to class of nonlinear differential equations. J Math Appl 2001; 4: 141-155.

Ongun MY. The Laplace Adomian Decomposition Method for solving a model for HIV infection of $CD4^{+}T$ cells. Mathematical and Computer Modelling 2011; 53: 597-603.

Rashidi MM, Erfani E. The modified differential transform method for investigating nano boundary-layers over stretching surfaces. Int J Numer Method H 2011; 21: 864-883.

Roohani Ghehsareh H, Abbasbandy S, Soltanalizadeh B. Analytical solutions of the slip Magnetohydrodynamik viscous flow over a stretching sheet by using the Laplace Adomian decomposition method. Z Naturforsch 2012; 67: 248-254.

Samadpoor S, Roohani Ghehsareh H, Abbasbandy S. An efficient method to obtain semi-analytical solutions of the nano boundary layers over stretching surfaces. Internat J Numer Methods Heat Fluid Flow 2013; 23(7): 1179-1191.

Schlichting H. Boundary Layer Theory. 6th edition, McGraw-Hill, New York 1979.

Sivakumar TR, Baiju S. Shooting Type Laplace–Adomian Decomposition Algorithm for nonlinear differential equations with boundary conditions at infinity. Applied Mathematics Letters 2011; 24: 1702-1708.

Soltanalizadeh B, Roohani Ghehsareh H, Yildirim A, Abbasbandy S. On the Analytic Solution for a Steady Magnetohydrodynamic Equation. Z Naturforsch 2013; 68: 412-420.

Van Gorder RA, Sweet E, Vajravelu K. Nano boundary layers over stretching surfaces, Commun. Nonlinear Sci Numer Simul 2010; 15: 1494-1500.

Wang CY. Flow due to a stretching boundary with partial slip—an exact solution of the Navier–Stokes equations. Chem Eng Sci 2002; 57: 3745-3747.

Wang CY. Analysis of viscous flow due to a stretching sheet with surface slip and suction. Nonlinear Analysis: Real World Applications 2009; 10: 375-380.

Wazwaz A. The combined Laplace transform-Adomian decomposition method for handling nonlinear Volterra integro-differential equations. Appl Math Comput 2010; 216: 1304-1309.

White FM. Viscous fluid flow. 3rd edition, McGraw–Hill, New York 2006.