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ORIGINAL ARTICLE

DETERMINING EMPIRICAL EQUATIONS FOR BED LOAD CALCULATION IN INTERNATIONAL SYSTEM OF UNITS (SI)

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ABSTRACT: Considering variable experimental and natural conditions, different equations for bed load calculation have been obtained by different researchers. According to conditions dominating on rivers, these equations show different results. The problem with most of these equations is that they are empirical and using them, considering that they are obtained in different systems, is not easily feasible and even a small mistake when using them will generate wrong results. In this research, in addition to collecting some of the most popular equations used by researchers, imposing changes on units and implementing related coefficients, the whole equations will be based on international system (SI).

KEYWORDS: Bed Load, Shields Equation, Hydrodynamic Power, Discharge, Critical Stress.

INTRODUCTION

In case, particles motion threshold surpasses their critical motion, in order to move, they will leap, roll and slide. Generally, bed load transfer rate in a river is 5-25% suspending load (Yang, 2001). Since, direct evaluation of bed load is not possible in most cases; many scientists provide variable equations based on hydraulic data, section properties, sediment properties etc. (Camenen and Magnus, 2005). A large amount of these equations is empirical and obtained in different units. This is why accurate dimensions must be considered when using existing equations; otherwise, wrong results will be produced. In this research, all the equations provided are obtained in IS and eventually, a computer program is written in FORTRAN language where applying basic information, bed load level is calculated on the basis of kilogram in different methods and will be accessible if needed (Shafaii Bajestan, 2008).

MATERIALS AND METHODS

As was mentioned, in most cases, direct bed load calculation is not feasible and empirical methods are required. Therefore, looking though major articles and books on bed load calculations, some of these equations are exploited and changing them, equations similar in dimension are obtained (Hamzepouri, 2006b). In the end, a computer program was written in FORTRON language. It is worth noting that this research will continue and more works will be prepared in future.

2.1. Calculating bed load

As was mentioned, most of existing equations for bed load calculation are empirical. In addition to units considered in each formula, the other problem with these equations is that each equation was obtained considering certain experimental data or a limited number of field data for particular conditions of the river. So, the accuracy of these equations should be evaluated in order to use them for different rivers (Afzalimehr *et al.*, 2005; Hamzepouri, 2006a). Resembled empirical equations are as followed:

2.1.1. Duboy and Straub equation

Duboy provided following equation in 1879 based on the hypothesis that bed materials move like fluid in parallel layers:

$$q_b = \psi.\tau_0(\tau_0 - \tau_c) \tag{1}$$

Straub provided ψ and τ_c values based on experimental data:

$$\psi = \frac{0.173}{D_{50}^{0.75}} \tag{2}$$

$$\tau_c = 0.0125 + 0.019D_{50} \tag{3}$$

Where, D_{50} is middle size of particles on the basis of mm, Ψ is Duboy coefficient on the basis of $\frac{ft^3}{s.lb}$, $\tau_c$is section critical stress on the basis of $\frac{lb}{ft^2}$. Results from these equations show that τ_c is also obtainable through Shilds diagram (Yang, 2001). Implementing related coefficients and placing them in equation (1), Duboy- Straub equation will be as followed:

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$$Q_b = 3.9484 \times 10^{-8} \gamma RS \frac{B\rho_s}{D_{50}^{0.75}} [\gamma RS - 909.091 D_{50} - 0.5981] \qquad (4)$$

Where, D_{50} is middle size of materials on the basis of m, γ is specific weight of water on the basis of $\frac{N}{m^3}$, R is hydraulic radius on the basis of m, S is ground gradient, B is the width of river ground on the basis of m, ρ_s is specific mass of sediment particles on the basis of $\frac{Kg}{m^3}$ and Q_b is bed load rate on the basis of $\frac{Kg}{s}$.

2.2.2. Casey equation

Based on experimental data, Casey provided two equations to evaluate bed load. Once coefficients are implemented, equations will be as followed:

A) Even sand bed

$$Q_b = 0.367 B \rho_s s_f^{\frac{9}{8}} (q - q_c) \qquad (5)$$

$$q_c = 1.633 \frac{D_{50}^{1.8}}{s_f^{1.2}} \qquad (6)$$

B) Uneven sand bed

$$Q_b = \frac{1}{3} B \rho_s S_f (q - q_c) \qquad (7)$$

$$q_c = 1.156 \times 10^{-3} \frac{D_{50}^{0.75}}{s_f^{1.25}} \qquad (8)$$

Where q is discharge in river width unit on the basis of $\frac{m^3}{s.m}$ and q_c is discharge in critical width unit of current on the basis of $\frac{m^3}{s.m}$ which are evaluated through (6) and (8) equations, considering the conditions. D_{50} is middle size of sediment particles on the basis of m and s_f is energy line gradient.

In the following, with all units changed into SI, repeated parameters are not included and just new ones are introduced.

2.2.3. Shilds equation

Shilds equation is obtained based on experimental data. The experiments are carried out in a 40×80 flume using five kinds of 1.7-2.5 m sediments and 1.06- 4.2 standard deviation of particles. A equation without Shilds dimension will be as followed. It is worth noting that, due to dimensionless nature of this equation, it is applicable in different units.

$$Q_b = 10 B.q.s_f \frac{\tau_0 - \tau_c}{(G_s - 1) g.D_{50}} \qquad (9)$$

Where, τ_0 is existing section stress values and τ_c is existing critical section stress evaluated in the following manner. G_s is relative density of particles.

$$\tau_0 = \gamma.R.S \qquad (10)$$

$$\tau_c = 0.056 (G_s - 1) \gamma.D_{50} \qquad (11)$$

2.2.4. Schoklitsch

Schoklitsch presented the discharge of river to evaluate bed load and provided two formulas in 1934 and 1943.

A) Equation presented in (1934)

$$Q_b = 221.36 B \frac{S^{\frac{3}{2}}}{D_{50}^{\frac{1}{2}}} (q - q_c) \qquad (12)$$

$$q_c = 0.00001944 \frac{D_{50}}{S^{\frac{4}{3}}} \qquad (13)$$

B) Equation presented in (1943)

$$Q_b = 2500 B.S^{\frac{3}{2}} (q - q_c) \qquad (14)$$

$$q_c = 0.6 \frac{D_{50}^{\frac{3}{2}}}{S^{\frac{7}{36}}} \qquad (15)$$

2.2.5. Meyer – Peter equation

The equation is obtained based on experimental data using five kinds of 28.6- 3.1 sediments, recommended for sand rivers. Once changes are implemented, the equation will be as followed:

$$Q_b = B \left[2.5 S (\rho.q)^{\frac{2}{3}} - 42.5 D_{50} \right]^{1.5} \qquad (16)$$

2.2.6. Meyer - Peter – Muller

The formula is best for bed load evaluation and it is obtained when suspended load does not exist so it is used for rivers with excessive bed load and least suspended load. After changes are implemented, Meyer- Peter- Muller equation is as followed:

$$Q_b = 1000 B \left[\frac{4.08 \times 10^{-3} \gamma R S_r - 1.9176 \times 10^{-4} \gamma (G_s - 1) D_{50}}{\left(\frac{\rho}{g}\right)^{\frac{1}{3}}} \right]^{\frac{3}{2}} \qquad (17)$$

Where S_r is the resistance resulted from particles and it is evaluated using following equation:

$$S_r = \frac{V^2}{K_r^2.R^{\frac{4}{3}}} = \left(\frac{q}{y.K_r.R^{\frac{2}{3}}} \right)^2 \qquad (18)$$

$$K_r = \frac{26}{D_{90}^{\frac{1}{6}}} \qquad (19)$$

Where V is current speed on the basis of m/s, K_r is Maning coefficient opposite and y is current depth on the basis of m.

2.2.7. Frijlink equation

Once changes are implemented on the following equation, in SI, it will appear as followed:

$$\mu = \left(\frac{Log\,\frac{12y}{D_{50}}}{Log\,\frac{12y}{D_{90}}} \right)^{\frac{3}{2}} \qquad (20)$$

$$\theta = \frac{\tau_0}{(G_s - 1)\gamma D_{50}} \qquad (21)$$

$$u_* = \sqrt{gRS} \qquad (22)$$

$$Q_b = 5B.\rho_s.\mu^{0.5}.u_*.D_{50}.e^{\frac{-0.27}{\mu.\theta}} \qquad (23)$$

Where, μ is dimensionless effect factor of ground shape, D_{90} is the diameter of 90% sediments transferring and u_* is section speed on the basis of *m/s.*

2.2.8. Rottner equation

The equation is obtained based on regression analysis of experimental data. Considering dimension, it is homogenous and applicable in any unit system.

$$Q_b = B.\rho_s[(G_s - 1)gy^3]^{\frac{1}{2}} \times \left[\frac{q}{y[(G_s - 1)gy]^{\frac{1}{2}}} \times \left[0.667 \left(\frac{D_{50}}{y} \right)^{\frac{2}{3}} + 0.14 \right] - 0.778 \left(\frac{D_{50}}{y} \right)^{\frac{2}{3}} \right]^3 \qquad (24)$$

2.2.9. Talin equation

If $\theta_c = \frac{\tau_c}{(G_s - 1)\gamma D_{50}} = 0.047$, then Q_b will be as followed:

$$Q_b = B.\rho_s.\theta^{\frac{1}{2}} F_y(\theta - 0.047)\sqrt{g(G_s - 1)D_{50}^3} \qquad (25)$$

$$F_y = 13.511 \left[1 - \frac{1}{\alpha T} Ln(1 + \alpha T) \right] \qquad (26)$$

$$\alpha = 0.5311 G_s^{-0.4} \qquad (27)$$

$$T = \frac{\theta}{0.047} - 1 \qquad (28)$$

$$\theta = \frac{\tau_0}{(G_s - 1)\gamma D_{50}} \qquad (29)$$

Where, T is dimensionless parameter of ground stress, θ is Shilds dimensionless parameter and θ_c is Shilds critical dimensionless parameter.

2.2.10. Bagnold equation

After coefficients are implemented, the equation will be as followed:

$$Q_b = \frac{e_b.\rho_s.q.(1 + S^2)}{\gamma.(G_s - 1)(0.6 - S)} \qquad (30)$$

Where, e_b differs from 0.1 to 0.2.

2.2.11. Van- Rijn equation (1984)

Van –Rijn noted that in bed load movements, gravity and hydrodynamic forces are dominating, and particles leap. For each sediment particle, he wrote movement size equation and obtained following equation:

$$D = D_{50} \times \left(\frac{g(G_s - 1)}{V^2} \right)^{\frac{1}{3}} \qquad (31)$$

$$C = 7.8173 Log \frac{4y}{D_{90}} \qquad (32)$$

$$\tau^* = \gamma \left(\frac{q}{y.C} \right)^2 \qquad (33)$$

$$\tau_c = 0.056(G_s - 1).\gamma.D_{50} \qquad (34)$$

$$T = \frac{\tau^* - \tau_c}{\tau_c} \qquad (35)$$

If $T < 3$:

$$Q_b = 0.053B.\rho_s.D^{-0.3}.T^{2.1}.\sqrt{(G_s - 1).g.D_{50}^{\frac{3}{2}}} \qquad (36)$$

If $T > 3$

$$Q_b = 0.1B.\rho_s.D^{-0.3}.T^{1.5}.\sqrt{(G_s - 1).g.D_{50}^{\frac{3}{2}}} \qquad (37)$$

2.2.12. Nielsen equation

Providing some equations, Nielsen calculated bed load. Combining these equations, bed load is evaluated directly.

$$Q_b = 12\,B.\rho_s(\theta - \theta_c).\sqrt{(G_s - 1).g.D_{50}^3.\theta} \qquad (38)$$

Considering Shilds critical stress, θ_c value can be 0.056.

2.2.13. Zhang and McConnachie (1994)

$$Q_b = \frac{9.3B.\rho_s}{\beta} \times \frac{u_*.D_{50}}{\sqrt{\theta}} (\theta - \theta_c)\left(\sqrt{\theta} - 0.7\sqrt{\theta_c}\right) \qquad (39)$$

$$\theta = \frac{\tau_0}{(G_s - 1)\gamma D_{50}} \qquad (40)$$

Considering Shilds critical stress, θ_c can be 0.056.

2.2.14. Ribberink equation

$$Q_b = 11B.\rho_s(\theta - \theta_c)^{1.65}.\sqrt{(G_s - 1).g.D_{50}^3} \qquad (41)$$

Considering Shilds critical stress, θ_c can be 0.056.

2.2.15. Camenen and Larson equation

He obtained following equation for bed load calculation.

$$\varphi = 12\theta^{1.5} \exp\left(-4.5 \frac{\theta_c}{\theta} \right) \qquad (42)$$

$$\theta = \frac{q_b}{(G_s - 1)g.D_{50}^3} \qquad (43)$$

He, also, considered that $\theta_c = 0.04$. Placing these equations in each other and θ_c, the following equation is obtained for bed load calculation.

$$Q_b = 12. B. \rho_s. \theta^{1.5}. \sqrt{(G_s - 1). g. D_{50}^3} \times \exp\left(-\frac{0.18}{\theta}\right) \tag{44}$$

RESULTS AND DISCUSSIONS

Empirical equations used in bed load calculations response in particular conditions and when using them, great care is required to maintain equation conditions. Equations obtained in this article have the advantage of being in one system, which is SI and this results in their ease of application (Xiaofeng and Mcconnachie Gordon, 1994; Mutlu Sumer *et al.*, 2003). It should be noted that before empirical equations are used, their accuracy level for a particular river should be examined, the issue that will be considered in future works.

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