

EVALUATION OF MATHEMATICAL MODELS TO ESTIMATE THE TRANSVERSE VELOCITY DISTRIBUTION OF THE SECONDARY FLOW COEFFICIENT

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ABSTRACT: To obtain the velocity distribution in Compound channels numerical method two-dimensional SKM for the based Navier-Stokes equation has been used. Combined with the roughness of the channel strip in bed, generally due to strong secondary flows caused by changes in bed roughness distribution, existing methods for estimating the velocity distribution of error rates are high compared to the actual results. In this study, using the above method and comparison with the experimental results shown the second parameter of the two-dimensional method, the maximum error in the two-dimensional model of the composite cross section at 0.004% for the fitted values of the rate the experimental data is measured. Comparison of flow curves is shown scale with which the level of variable roughness due Non-homogeneous flood plains streams do not interfere with the depth of the secondary flow coefficient increases the error in the calculations is 21 percent, which increases with depth, the greater the error be.

Keywords: secondary flow, SKM 2D-dimensional model, the velocity distribution.

INTRODUCTION

The rivers are one of the natural phenomena, which have inflicted numerous losses and defects to the human since long time ago. Due to the special influence that they had in human life and emerging of various civilizations, they have always invited them to utilization from water as a bounty and living in lands on the bank of the rivers. Most of rivers have so far created many problems with respect to geographic, geologic, and climatic situation in several zones with seasonal flows and streams caused by floodplains. The group of these problems clearly identifies the necessity for taking the essential measure on the rivers. Today, water consumption has been increased extremely with developing the cities and using gravitational force and moving of water as a flow at free level in channels may be considered as the most prevalent techniques in water transfer, irrigation, collection and transfer of wastewater

or ground waters. For this reason, some measures like identifying and recognition of various quantities such as average speed profile in depth, distribution of transverse speed, secondary discharge and flow in cross-section of the water flow are crucially important to prevent from possible risk.

Due their impact on flow hydraulic property, the review of secondary flows is important. Despite of the little ratio of secondary flow to basic streams, their study is vitally important because of the flow resistance, especially in their Compound Channels [1]. The flow is laterally transferred from the main channel to floodwater extents and this may increase the capacity for transfer upon occurrence of floodwater [2]. According to Fig (1), the Compound Channels are the combination of a Main Channel and wide floodplains, which are often dried during a year and they possess higher roughness coefficient than main channel.

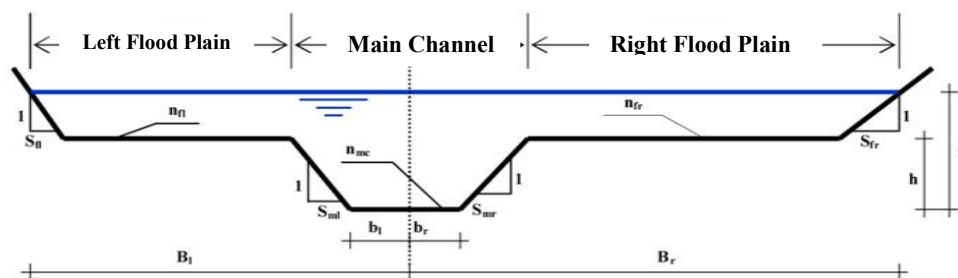


Figure 1 -schematic of the compound channel with the floodplain zone [1]

When the flood occurs, it fills out the main channel and enters into floodplains. Under this condition, due

to difference in depth of flow and roughness coefficient among the main channel and floodplains a notice resistance is created in their connection boundary. Under floodwater condition, cross-section of river usually comprises of a main channel and two floodplains in which the flow is deep with high speed at main channel while the flow is shallow with low speed in the floodplain. Thus, division of river into the main channel and floodplains is the paramount phase in flow hydraulic computations in Floodwater Rivers [3]. This flow mainly originates from difference in shear stress among fluidal layers. Due to

difference in flow speed among the main channel and floodwater bank in the Compound Channels, shear stress is created among the fluid layers and it causes formation of secondary flows in this zone [4]. The remarkable difference in flow speed between floodplain and main channel may cause an interactive zone in the border among these two regions and it leads to exchange the mass, momentum, and noticeable energy loss on them [5]. Fig (2) displays the place of entering the main channel into floodplain and effect of secondary flow on the transverse speed that has been created due to mutual effect of the flow [6].

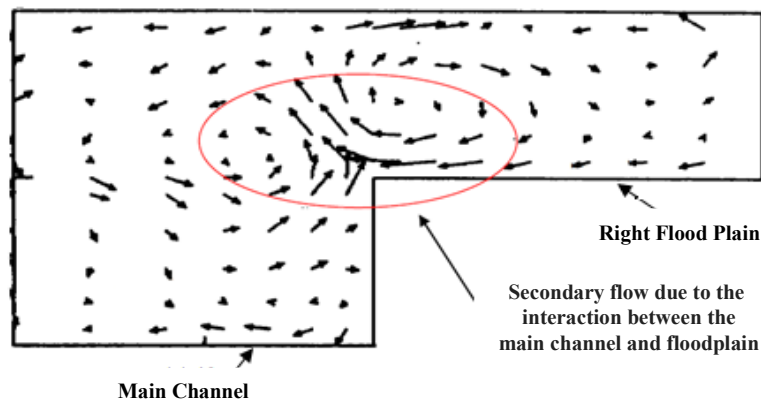


Figure 2- Secondary flow between the main channel and floodplain sections [6]

The flow in the compound channels has been analyzed in hydraulic references in such a way that initially the composite section is divided into partial sections (main channel and floodplains) and then flow discharge is computed in each of these sections by means of Manning Equation. Finally, total discharge will be obtained from sum of the partial discharges. While, the results of various researchers indicate that the acquired flow discharge by this technique includes up to 40% error compared to the real value [7].

RESEARCH HISTORY

Sellin (1964) expressed the phenomenon of mutual effect and reduction of discharge due to creation of turbulence among subsections for the first time [8]. Since that time, a lot of studies have been conducted regarding the adjustment of the common techniques for calculation of discharge with respect to momentum transport. Of the paramount samples of these studies, it was the establishment of a well-equipped

laboratory center in Hydraulic Research, Wallingford in 1985. Creation of such utilities was aimed at recognition of flow hydraulics and sediments in Compound Channels by means of accurate laboratory data.

The researchers have purposed several techniques for intervention of momentum transfer in the relevant calculations to the Compound Channels by means of laboratory results in order to adjust flow hydraulics in them. The results of these studies have led to noticeable advancements in calculation of discharge- scale of laboratory Compound Channels and fluvial sections. These techniques including one- dimensional, 2D, and 3-D methods are called adjusted techniques [9].

Since the above adjusted techniques have been presented by assuming under conditions of viable and uniform flow thus their application cases have been also noticed (e.g. discharge-scale equation, calculation of transverse speed and boundary shear strength, determination of critical depth, and estimation of sediment transport) under the same flow conditions. The

other important applications such as water level profile and tracking floodwater trend may be studies in Compound Channels so they have been still rarely noticed. Most of the conducted studies were related to laboratory channels with composite section while the hydraulic and river-related engineers conduct their studies for better management of floodplains in natural rivers. The review of Shiono and Knight Method - SKM in Compound Channel is the important point in the present investigation. In 1988, 2-D analytical model was purposed based Navier-Stokes Equations to solve the transverse distribution of speed and shear stress in simple and Compound Channels. The effects of secondary flows were ignored in this mathematical model [10]. The results of this mathematical model in laboratory and fluvial Compound Channels showed that the secondary flows might play important role especially in determination of transverse distribution of shear stress. The numerical solution of this two-dimensional model has been also presented by means of limited difference [3] and finite element methods [11]. Application of these two numerical techniques in tracking floodwater trend in homogeneous Compound Channels has been purposed totally identical results [12]. In 1999, a one-dimensional technique was purposed for Exchange Discharge Method (EDM) in main channel and floodplains by Bousmar et al. in this method, the values of factor of transfer in main channels and floodwater are corrected by means of appropriate adjusted coefficients and flow discharge is computed with suitable accuracy. This method is appropriate used in determination of hydraulics of irregular Compound Channels and calculations of water level profile [13].

Ervin et al (2000) have purposed 2-D analytical method based on Navier-Stokes Equations for direct and labyrinth Compound Channels [14]. It has been show in this study that the effects of secondary flow in Compound Channels with labyrinth path are approximately ten times greater than the corresponding flow in Compound Channels with straight course. By means of numerical solution of 2-D model [15] as well as Coherence - technique [7] a new method has been suggested for calculation of discharge- scale equations in regular and fluvial heterogeneous Compound Channels [16]. In this technique, the coefficient of current flows for main channel is expressed as a third-order function of the adjusted relative roughness in the mode of heterogeneous Compound Channels.

MATERIALS AND METHODS

There are a lot of two-dimensional hydraulic methods, which have been developed by SKM, Varek et al, Lambert and Sellin, Ervin et al, and Progen et al [18]. Several studies have been carried out on the existing data in floodwater channel by the aid of the above-said methods at large scale. By the aid of SKM method in this survey, in order to study on two-dimensional performance of secondary flow in Compound Channel, the acquired laboratory data from Birmingham University have been employed [1]. SKM technique has predicted side variations of depth mean speed in the Compound Channel by purposing an analytical solution for Navier-Stokes Equation. Navier-Stokes Equation can be assumed as an equation with a fluid element in the flow that is considered in combination of momentum equation with continuity equation:

$$\rho \left[V \frac{\partial u}{\partial y} + w \frac{\partial w}{\partial z} \right] = \rho g \sin \theta + \frac{\partial \tau_{yx}}{\partial y} + \frac{\partial \tau_{zx}}{\partial z} \quad (1)$$

(In terms of physical concept: Reynolds stress (transverse+ vertical) + Weight force = Secondary flows), where u , v , w denote local speed in x-axis of the flow, y- axis at width of flow, and z-axis in vertical direction of flow. $S_0 = \sin \theta$ is bed gradient, τ_{yx} and τ_{zx} are Reynolds stresses on the floor. Overall, whenever speed components v and w become important as transverse and vertical elements of flow speed, the secondary flow emerges and it remarkably effects on hydraulic properties of Compound Channels [4].

SKM technique has presented the mean speed equation in depth by integration of flow depth (H) on opposite viscosity flows [2] by ignoring the secondary flow according to Eq. (2) [18].

$$\rho g H S_0 - \frac{1}{8} \rho f U_d^2 \left(1 + \frac{1}{s^2} \right)^{1/2} + \frac{\partial}{\partial y} \left\{ \rho \lambda H^2 \left(\frac{f}{8} \right)^{1/2} U_d \frac{\partial U_d}{\partial y} \right\} = 0 \quad (2)$$

Where in the above equation: U_d denotes mean speed in depth (subscript d means average depth), ρ is water volumetric mass, g as gravitational acceleration, λ as dimensionless coefficient of viscosity (turbulent flow), which has been derived according to the data of turbulent flow and after determination of mean depth of Reynolds stresses, and f is dimensionless coefficient of friction in Darcy-Weisbach equation, H as depth of flow in any point, S_0 as floor gradient of the conduit, y transverse direction of conduit, and s Main Channel Lateral Side Slope [12]. Eq. (2) applies only when the secondary flows are not

considered [20]. In the case of intervention of effect of secondary flows, which are important, and given the very small value of vertical component of the flow speed (H-value is supposed as constant), differential Eq. (2) was adjusted as Eq. (3) by means of SKM technique in 1991.

$$\rho g H S_0 - \frac{1}{8} \rho f U_d^2 \left(1 + \frac{1}{s^2}\right)^{1/2} + \frac{\partial}{\partial y} \left\{ \rho \lambda H^2 \left(\frac{f}{8}\right)^{1/2} U_d \frac{\partial u_d}{\partial y} \right\} = \frac{\partial}{\partial y} [H(\rho UV)_d] \quad (3)$$

In the following equation, s is the main channel lateral side slope.

DATA USED FOR THE 2D-DIMENSIONAL MODEL

Several data are required to implement two-dimensional mathematical model in laboratory

sections and natural rivers. The majority of these data are of those kinds of data, which most of one-dimensional mathematical models need to them in implementation of river engineering projects. For this reason, these data exist in many rivers and especially at the site of hydrometric stations. The measured values of transverse distribution of flow speed are specific data for this mathematical model and they are measured by the aid of water flow-meter device (Moline) in rivers. To show the superiority of the suggested mathematical model in this investigation compared to all flow hydraulic models and to solve transverse speed distribution under floodwater and calibration of this model, the relevant data to flow speed should be employed under floodwater conditions. In Table (1), the needed data for execution and calibration of 2-D mathematical model are listed.

Table1- data and required Statistics for implementing 2D-dimensional mathematical model in rivers [19]

Information required	Stage used	
	Run Model	Calibration
Transverse sections of the river	✓	✓
The longitudinal slope of river or water surface slope	✓	✓
Manning roughness coefficient	✓	-
Manning scale river flow within the normal flow and flood	-	✓
Flow velocity distributions measured at various depths	-	✓

ANALYSIS OF SECONDARY FLOW COEFFICIENT

The main definition of K-value by this Literature is utilized to express the relationship $(UV)_d, U_d^2$ by assuming that the elements of transient mean speed (U,V) are the general elements of average speed [15]. Ikeda equation has purposed an analytical description for transverse speed of flow as Eq.(4) [16]:

$$\frac{V}{U_*} = \frac{6\delta}{k\pi^2} \text{Sin}\left(\frac{\pi y}{H}\right) \left[2\text{Cos}\left(\frac{\pi z}{H}\right) - \pi\left(\frac{2z}{H} - 1\right) \text{Sin}\left(\frac{\pi z}{H}\right) \right] \quad (4)$$

Where U^* denotes shear speed, k is Von Karman's Constant, and δ is turbulence range. Therefore, with respect to Eq. (4), transverse speed (V) may be both positive and negative. As a result, it can be found that mean depth $(UV)_d$ UV can be both positive and negative in which k is ratio coefficient.

$$\rho g H S_0 - \frac{1}{8} \rho f U_d^2 \left(1 + \frac{1}{s^2}\right)^{1/2} + H \frac{\partial}{\partial y} \left\{ \rho \lambda \left(\frac{f}{8}\right)^{1/2} H U_d \frac{\partial u_d}{\partial y} \right\} = \frac{\partial}{\partial y} [H(\rho UV)_d] \quad (5)$$

$\Gamma = \frac{\partial H(\rho UV)}{\partial y} = \frac{\partial}{\partial y} [H(\rho UV)_d] ; UV = k U_d^2$
 This constant is a function of depth and roughness of floodplain. With substitution of Eq. (6) with Eq. (5) then Eq. (7) may be changed into ordinary differential equation (7).

$$\rho g H S_0 - \frac{1}{8} \rho f U_d^2 \left(1 + \frac{1}{s^2}\right)^{1/2} + H \frac{\partial}{\partial y} \left\{ \rho \lambda \left(\frac{f}{8}\right)^{1/2} H U_d \frac{\partial U_d}{\partial y} - \rho k U_d^2 \right\} - \frac{\partial H}{\partial y} \rho k U_d^2 = 0 \quad (7)$$

With respect to analysis of above subsections, k-value is often negative in main channel and it is positive in floodplain. And even in some cases this value is greater than 0.50% that was suggested by Ervin et al [18]. if we always evaluate K-value as positive this assumption will not be often suitable. Value of $(UV)_d$ may be

positive if the flow move clockwise (along the flow path) and this value can be negative if the flow move counterclockwise. As it was shown by Liu et al this issue is also consistent with this study [20].

RESULTS AND DISCUSSION (COMPARISON OF MODEL CALCULATIONS AND EXPERIMENTAL DATA)

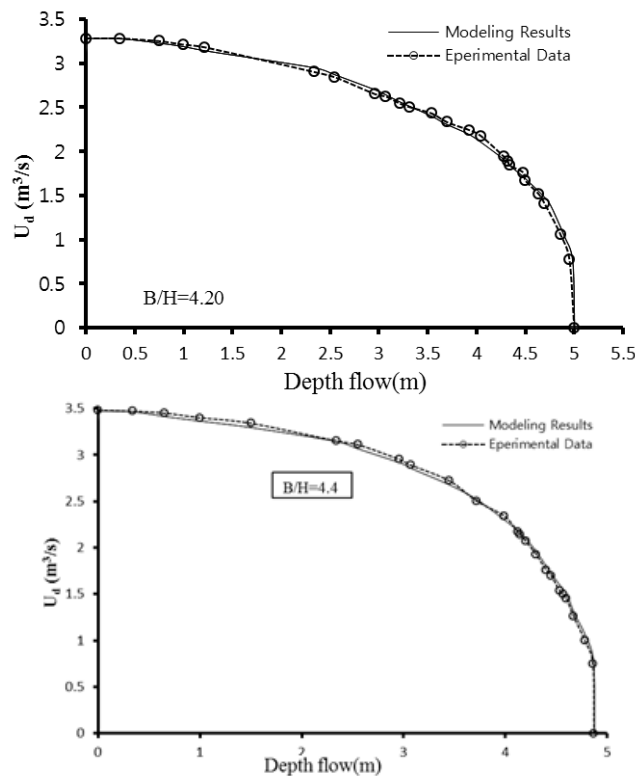
The Compound Channel has been shown with symmetric geometric figures in Fig (1) and geometric details of this system in Table (2). In Fig (3), the paramount parameter for calibration of mathematical models of flow hydraulics in

channels and geometry of section as well as specifications of flow speed in contrast to empirical data has been presented. The known data includes geometry of transverse section and figures of water level are considered as inputs in the suggested mathematical model. It requires noting that the values of flow speed distribution for empirical data at large scale have been derived from the conducted laboratory studies in Birmingham University. It is obvious that distribution of the derived speed from empirical data should be consistent with the results of speed transverse distribution from the suggested model in the composed channel in this investigation.

Table (2): Geometric details of eleven groups from the symmetric Compound Channel

Number of sets	S ₁	B/H	Type floodplain
FCF0106	1	6.67	Smooth
FCF0205	1	4.20	Smooth
FCF1005	2	4.40	Smooth

(B/h: Aspect Ratio for half of width of main channel to full depth of main channel)



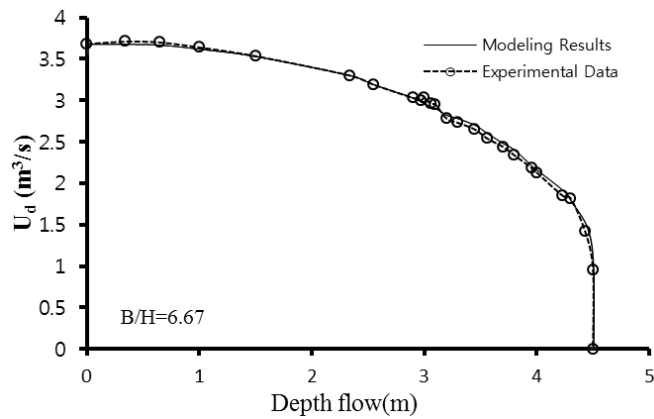
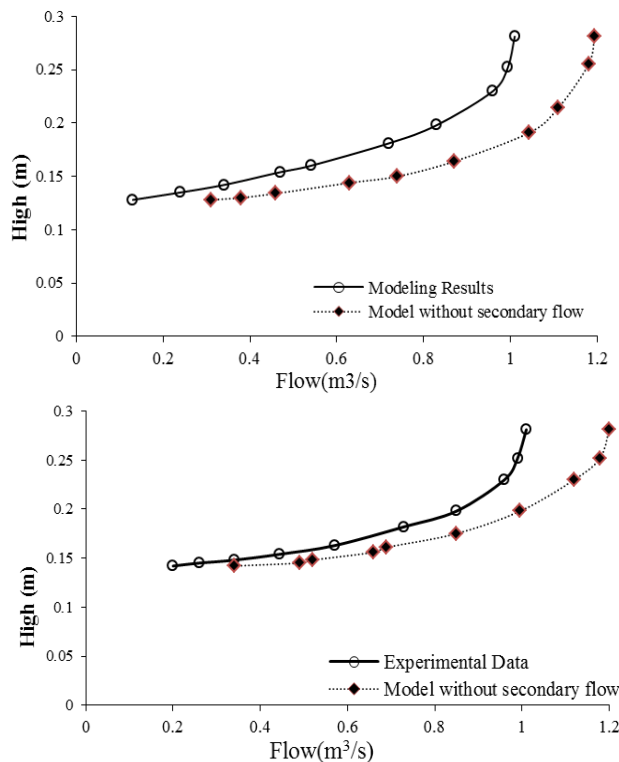


Figure 3- Comparison of the transverse velocity distribution of the proposed model with experimental data

With respect to Fig (3) it can be implied that proximity of the suggested model to results of the empirical data has caused the appropriate estimation from speed transverse distribution. The results signify that the maximum error in the suggested 2-D model has been derived about 4% in composite section for fitting speed value that is accepted for this model and the measured empirical data. With comparison of the acquired discharge-scale curves from both models, which

are shown in Fig (4) it is found that it has appropriate performance in homogeneous section and ignoring secondary flow was not followed by a lot of error but due to variable of roughness in floodplains with flow depth and lack of interference of secondary flow term in heterogeneous section, it increased error up to 21% in the given calculations and as depth increases, the flow becomes greater.



A- Non-homogeneous B- Homogeneous
Figure 4- Comparison of DEBI-ASHEL curves in compound channel

VALIDATION OF RELATIONS

One of the foremost properties of every mathematical model is its capability in

prediction of flow conditions appropriately after calibration phase. This phase is called validation. For this purpose, the regression curve for variance of this coefficient with respect to flow

depth has been drawn for main channel of flow and floodplains by means of the derived roughness coefficients from calibration. The linear regression curves are shown in Figs (5) with determination coefficient 0.9905 for main channel of flow as well as determination

coefficient 0.9932 for floodplains. With calibration of the suggested 2-D mathematical model by Abril [21], the ratio coefficient in main channel and floodplains for the laboratory Compound Channel with straight path were derived 0.020 and 0.043 respectively.

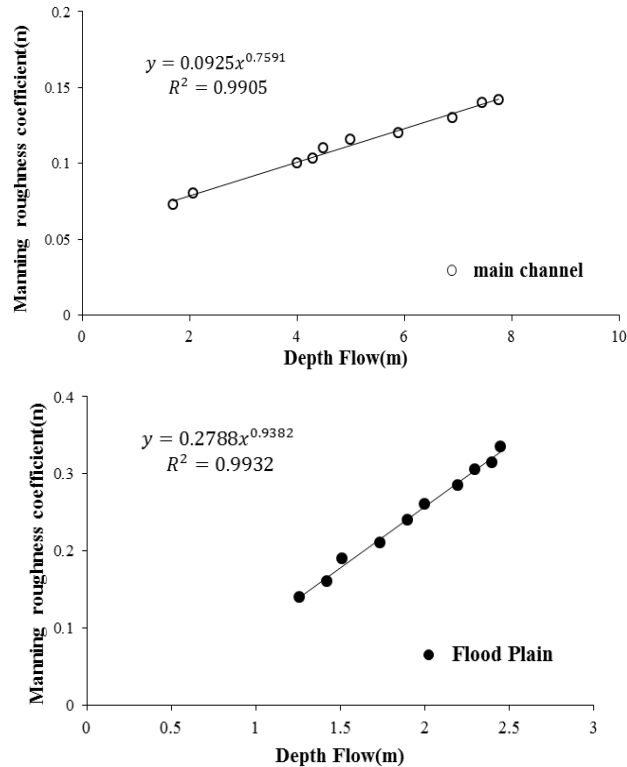


Fig (5): The relationship among Manning roughness coefficient and variance of flow depth in main channel and flood plains of flow

DEBI-ASHEL curves

The accuracy of prediction for transport capacity of channel with floodplain is the foremost unreliable part in analysis of rivers. With calculation of speed transverse distribution in main channel and floodplain of river, the better grounds will be prepared for floodwater management, distribution of pollutants, and transport of fluvial suspended sediments. With

respect to the acquired results from speed transverse distribution at any depth of flow, flow discharge can be computed by numerical integration. In Fig (6), some calculations are shown by means of 2-D model with discharge-scale curve that indicates high accuracy of 2-D model in estimation of flow discharge even under floodplain conditions.

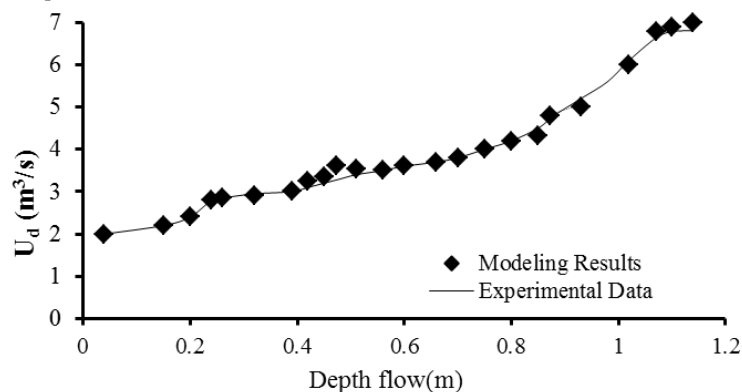


Figure 6- Comparison of calculating DEBI with using a 2D-dimensional model with DEBI-ASHEL curve

ASSESSING ERROR RESULTS OF 2D-DIMENSIONAL MATHEMATICAL MODEL

To make sure of proper performance of the model after preparation of numerical model and to examine the acquired results from numerical solution of mathematical model in calculation of speed transverse distribution and flow discharge the errors of results were evaluated. According

to Fig (7) error percentage of flow discharge and Fig (8) the comparison of speeds in depth 5.1m, 2-D mathematical model in this study possesses better accuracy for calculation of flow speeds in main channel and floodplains. 2-D method has less dispersion compared to bisection line (45°). To evaluate these results more accurately, determination coefficient (R^2) has been computed for the model.

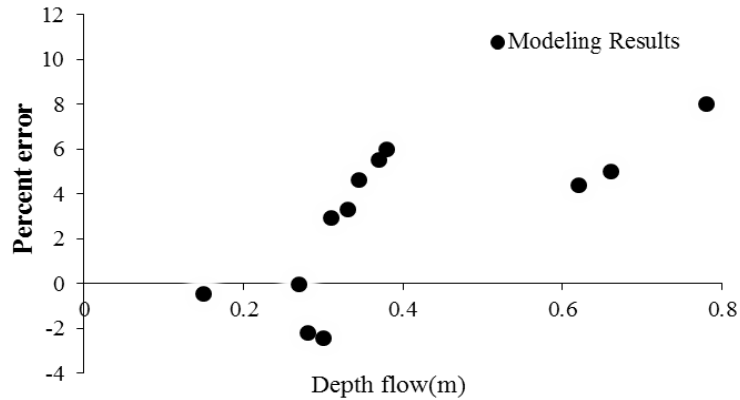


Figure 7- Comparison of calculating flow DEBI error Percent

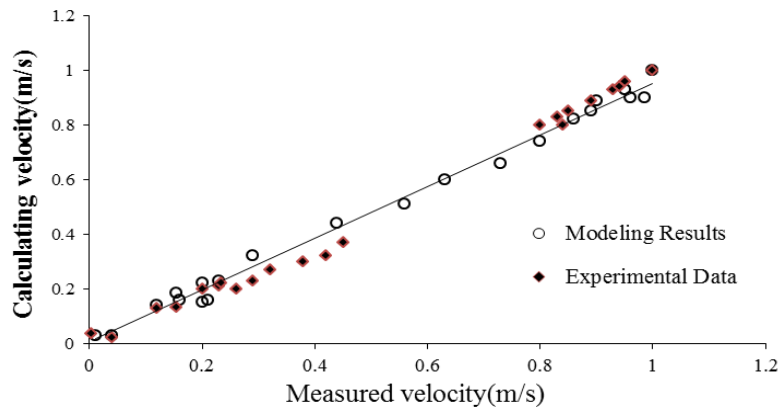


Figure 8- Comparison of the measured velocity than calculating velocity, at a depth of 5.1 m.

CONCLUSIONS

With respect to the acquired results from 2-D model in estimation of speed transverse distribution and their comparison with the results of empirical data, one can refer to the following results:

The comparison of the computed values in 2-D technique in the suggested model with empirical data in estimation of speed transverse distribution (mean speed in depth) with coefficient of secondary flow signifies the accuracy of performance of 2-D mathematical model in these channels.

One could refer to some advantages of 2-D mathematical model including evaluation of secondary flow coefficient in estimation of flow

discharge at any depth of flow by means of speed transverse distribution profile that has presented reasonable results in estimation of discharge-scale curve.

With respect to variance of roughness coefficient within channel width as well as variations in flow depth it was not possible to consider a fixed roughness coefficient thus given the existing relationship among changes in flow depth as roughness coefficient for several discharge-scale curves and for main channels of flow and floodplains and with considering the least error in estimation of the calculated and observed speed transverse distributions, a relationship was derived for computation of Manning roughness coefficient in main channel of flow and floodplains with linear variance with respect

to depth that might have suitable performance for other several discharge- scale curves in calculation of speed transverse distribution and flow discharge.

In addition to introducing various theories from several researchers for intervention of mutual effects of sections in this study, Theory of Ervin - Knight was employed in homogeneous and heterogeneous composite section. The comparison of the given results from the suggested mathematical model with empirical results from speed transverse distribution showed that the aforesaid theory might improve the results of the model to great extent. Likewise, the results of discharge- scale curve in these sections indicated that lack of intervention of effect of secondary flows in homogeneous composite section had not created error in these computations and the derived results were highly consistent with the laboratory values.

But the secondary flows have played very important role for heterogeneous composite section and it is recommended to consider these sections in analysis of flow. Ignoring the effect of secondary flow in the homogenous laboratory Compound Channels may maximally create 0.76% errors in calculation of flow total discharge while the maximum rate of error is about 21% in heterogeneous laboratory Compound Channels.

This study has analyzed the secondary flow with geometric figures based on the best process with empirical data SERC-FCF and finally the following results can be extracted from this analysis:

- (i) As depth is added in floodplain, speed distribution becomes more homogeneous at transverse section and coefficients of secondary flow are gradually increased.
- (ii) K- value (k in percent's) increases in floodplain with reduction in width of floodplain while K-values and rate of lateral gradient are gradually decreased in main channel.
- (iii) When floodplain is rough the speed variance becomes greater among the main channel and floodplain. Thus, coefficients of secondary flow become higher at transverse section.
- (iv) The K-values are often negative in main channel while they are positive in floodplain and even in some cases this value is greater than 0.50%. This phenomenon expresses that this assumption ($0 < K < 0.50$), which has been suggested by Ervin et al, is not always reliable.

The above results signify that it is crucially

important to determine K-value in any area for modeling of speed distribution in open Compound Channel.

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